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Innovative Applications of O.R.

Analytical solutions to the dynamic pricing problem for time-normalized revenue

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ABSTRACT

In this work we consider dynamic pricing for the case of continuous replenishment. An essential ingredient in such a formulation is the use of time normalized revenue or profit function, in other words revenue or profit per unit time. This provides the incentive to sell many items in the shortest time (and of course at a high price). Moreover, for most firms what matters most is how much revenue or profit is achieved in a certain time frame, for example per year. This changes the problem qualitatively and methodologically. We develop a new dynamic pricing model for this formulation. We derive an analytical solution to the pricing problem in the form of a simple-to-solve ordinary differential equation (ODE) equation. The trajectory of this ODE gives the optimal pricing curve. Unlike many of the models existing in the literature that rely on computationally demanding dynamic programming type solutions, our model is relatively simple to solve. Also, we apply the derived equation to two commonly used price-demand functions (the exponential and the power functions), and derive closed-form pricing curves for these functions.

1. Introduction

Dynamic pricing is the theory of adjusting the price of a merchandise dynamically with time in an attempt to maximize revenue (Elmaghraby & Keskinocak, 2003; Talluri & Ryzin, 2005 and den Boer, 2014). It has gained significant worldwide adoption in many industries in the last two decades. It is appealing that such an advanced science has found true application in many markets. The gains in the companies' revenues may also ultimately pass through to the customer, because pricing would more genuinely reflect supply and demand, leading to better economic efficiency (Phillips, 2005). What helped the spread of dynamic pricing is the rapid growth of internet retail. Its online nature makes the periodic updating of the price quite manageable. Moreover, dynamic pricing is expected to catch up soon in a big way for brick and mortar stores, due to the advent of electronic price labeling devices, whose cost became economic enough for possible wide-scale deployment (Stross, 2013).

Most research on dynamic pricing focuses on optimizing revenue, and sometimes profit. However, corporations consider not just these aspects. They focus on achieving the most amount of revenue or profit in the shortest time, for example achieving high annualized revenue or profit. In that sense turn-around is an important factor. In this work we propose a new dynamic pricing model that optimizes the revenue or profit per time period. This is more relevant to corporations, which typically target a specific amount of revenue by the end of their fiscal year or quarter (see Bouwens & Kroos, 2011). All numbers are either annualized, or else interpreted in an annualized way. So, the amount of revenue per year or per unit time is the relevant quantity. Moreover, having a time-normalized revenue as the objective function will likely encourage more sales in a fixed interval, thus achieving higher overall revenue in the interval.

The typical set up in the literature is to consider a perishable product with a particular stock that has to be sold by a certain time horizon (see for example Gallego & van Ryzin, 1994). In this work we allow continuous restocking. Thus, when an item is sold, it can be replenished immediately or with a certain lead time. This is the essential difference between our work and many of the other works that consider a fixed inventory. In our set-up of continuous replenishment it is the use of time normalized revenue (instead of pure revenue) which creates the urgency to sell many items. This is in contrast with the typical fixed inventory formulations, whereby the existence of the fixed inventory and a fixed time horizon provides the urgency to sell many items before times is up. The continuous replenishment set-up is also suitable for just-in-time inventory systems, and service-based businesses.

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But, as we will show, our model is also applicable to nonperishable goods with existing inventory, and in some situations to perishable items. In previous work in the literature on pricing of nonperishable items the time element is not considered a pressing consideration, and therefore the item could even be sold at a fixed price. However, when turn-around is considered as part of the objective, then a time varying price could control both the revenue and the timing of the sale in a way to achieve optimal revenue per unit time. This basically changes the problem qualitatively.

In this work we consider a continuous time formulation with stochastic demand. We assume that demand follows a Poisson process with a varying arrival rate. This is a sensible assumption, because of the random times of arrivals of the selling events. The instantaneous arrival rate varies according to the price-demand function, due to the continuous adjustment of the price. Moreover, we allow the case where the demand function varies explicitly with time (in addition to price). This is a useful case to consider, because for many products, even if nonperishable, interest wanes with time due for instance to the introduction of newer models. Most other work that considers stochastic demand led to computationally demanding solutions often involving complicated recursive relations for the value function (see Bitran & Caldentey, 2003). In contrast, our work leads to a solution for the general case in the form of an ordinary differential equation (ODE), which can be solved in a straightforward way using standard solvers such as Euler's method or Runge Kutta's method. Moreover, for two of the typical demand price functions, the exponential function and the power function, we derive closed form solutions for the optimal price curve (for example for the exponential demand function, the optimal price is of the form $(1/\alpha)\ln[a + b\exp(ct)]$ for some problem related constants α, b, and c).

The paper is organized as follows. In the next section, we discuss related work, and review the literature. Section 3 provides a statement of the problem and defines all relevant quantities. Section 4 provides our proposed solution, including the derived ODE. In Section 5 we apply the proposed model to two candidate price elasticity functions, and derive closed form solutions. Moreover, in this section we consider examples of time demand functions with explicit time variation. We also compare the proposed methods with existing benchmarks. Last section is the conclusion of the work.

2. Related work

The early work on dynamic pricing for perishable products considers deterministic demand models. This means that the seller has perfect knowledge about the demand process (Talluri & Ryzin, 2005). Since demand is inherently random, recent research considers stochastic models. Some researchers consider simple additive or multiplicative random terms on a multi-period demand function. More recently, researchers assume that demand follows a non-homogeneous Poisson process, where the arrival rate (or intensity function) is a function of the price. Embedded in this relation is the so-called reservation price, which is the maximum price the customer is willing to pay for the product. The pioneer for such an approach is Kincaid and Darling (1963), and it is further developed by Gallego and van Ryzin (1994). The solution for this formulation gives rise to an ordinary differential equation expressing the time evolution of the value function for different inventory levels. It is numerically very hard to solve this equation, except for some special cases, such as exponential reservation price (see Bitran & Caldentey, 2003).

Extensions of this modeling approach are proposed by Zhao and Zheng (2000) and by Anjos (2005) for the case where the arrival process of customers is a time-dependent Poisson process. This is typically encountered in airline reservations, where price-sensitive customers arrive earlier than high-paying customers, and in fashion retail where the opposite is true. Lazear (1986) considers a situation where the selling horizon consists of two periods with distinct prices, and obtains the break point value and the two prices that optimize revenue. Feng and Gallego (1995) consider a two-price model, where the seller optimally switches from one price to the other. Bitran and Mondschein (1997) consider the case where the price curve is piecewise constant, while Netessine (2006) tackles the situation where it is required to minimize the frequency of price changes. Recognizing that the demand characteristics are typically not accurately known beforehand, recent work focuses on learning the demand parameters online, hand in hand while applying dynamic pricing using the hitherto learned information (Araman & Caldentey, 2006; den Boer & Zwart, 2014; Petruzzi & Dada, 2002, and Farias & Roy, 2010). A more general direction is to consider dynamic pricing of multi-products. The difficulty of this approach is having to deal with a "vector" of prices, and at the same time having to take into account the cross-product influences (see Gallego & van Ryzin, 1997 and Rusmevichientong, Roy, & Glynn, 2006).

Some work considers dynamic pricing coupled with replenishment decisions. This basically integrates the fields of dynamic pricing and supply chain analysis. Chen and Simchi-Levi (2004a,b), and Federgruen and Heching (1999) propose a solution for this problem for the deterministic case, where continuous replenishment (every period) takes place. Li and Zheng (2006) consider the stochastic case. Specifically, they propose a joint dynamic pricing/replenishment model, where in addition to stochastic demand the production yield is stochastic too. Smith and Achabal (1998) consider the situation where demand is both seasonal and is dependent on the initial inventory. A comprehensive review of models that consider the interplay between dynamic pricing and replenishment can be found in (Chan, Shen, Simchi-Levi, & Swann, 2004).

There is little research that considers other objective functions than revenue or profit. Arcelus and Srinivasan (1987) consider three different kinds of target functions: profits, return on investment, and residual income (i.e. income net of interest payments). They use a derivative based solution method using a deterministic demand assumption. In a follow-up article Arcelus and Srinivasan (1988) analyze the sensitivity and the analytical properties of the solutions pertaining to these novel objective functions. Optimality conditions for the proposed solutions are proven, and an extensive graphical analysis for the effects of parameter changes is presented. Besbes and Maglaras (2012) consider dynamic pricing in the presence of intermediate sales targets at certain milestones. These are prevalent for real estate sales, as they are closely tied to the ability to obtain financing. They propose a model that is of a feedback form, where the price is continually updated so that it tracks the most stringent among all upcoming milestones. Bayoumi, Saleh, Atiya, and Aziz (2013) and Zakhary, Atiya, El-Shishiny, and El Gayar (2011) recognize the need for multi-objective formulations, so they suggest a flexible simulation-based approach. Most of the models in the literature assume that the seller is risk neutral. However, many product managers in charge possess some degree of risk aversion. To address this issue, Feng and Xiao (1999) consider risk as an additional measure that has to be taken into account. Their model is based on maximizing the revenue, while minimizing the variability in the revenue stream. Other studies that consider risk include the work of Choi, Chow, and Xiao (2012) and Sun and Abbas (2013). From the aforementioned review one can observe that most work considers optimizing revenue. There is little work on optimizing other performance measures. As far as we know there is no work on optimizing the amount of revenue of profit per unit time, or annualized revenue or profit, even though this is a major performance yardstick for