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Interfaces with Other Disciplines

Applied cost allocation: The DEA–Aumann–Shapley approach

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ABSTRACT

This paper deals with empirical computation of Aumann–Shapley cost shares for joint production. We show that if one uses a mathematical programing approach with its non-parametric estimation of the cost function there may be observations in the data set for which we have multiple Aumann–Shapley prices. We suggest to overcome such problems by using lexicographic goal programing techniques. Moreover, cost allocation based on the cost function is unable to account for differences between efficient and actual cost. We suggest to employ the notion of *rational inefficiency* in order to supply a set of assumptions concerning firm behavior. These assumptions enable us to connect inefficient with efficient production and thereby provide consistent ways of allocating the costs arising from inefficiency.

1. Introduction

Aumann–Shapley (A–S) cost allocation, often interpreted as generalized average cost sharing, is a well-known cost allocation method designed for regulation of multi-product natural monopolies as well as for internal cost accounting and decentralized decision making in organizations, see e.g., Spulber (1989), Banker (1999), Mirman, Tauman, and Zang (1985a). In short, the idea is to determine a set of unit prices for each output, i.e., the Aumann–Shapley (A–S) prices, and use these for the allocation of joint costs.

The theoretical literature has shown that the A–S method (and A–S prices) possesses a number of desirable properties, see e.g., Billera and Heath (1982), Mirman and Tauman (1982), Young (1985), Mirman, Tauman, and Zang (1985b), and it has essentially been the unanimous recommendation of economists for decades when sharing the costs of joint production, see e.g., Friedman and Moulin (1999). Yet, despite its sound theoretical foundation there has been relatively few empirical applications. The reason seems at least twofold:

1. It requires an empirical estimation of the cost function that enables computation of all relevant A–S prices.
2. In practice firms may not produce at efficient production cost. Hence, an allocation based solely on the cost function will not account for differences between efficient and actual costs.

In the present paper we examine how to cope with both these issues. While there has been previous papers dealing with computation of A–S prices for given empirical cost functions we believe the second issue, concerning inefficient production, has been ignored and we offer a completely new approach here.

We follow up on papers by Samet, Tauman, and Zang (1984), and Hougaard and Tind (2009) and consider empirical estimation based on convex envelopment of observed cost-output data as in the celebrated Data Envelopment Analysis (DEA) approach of Charnes, Cooper, and Rhodes (1978).[1] The resulting piecewise linear cost function enables a relatively simple computation of A–S prices for large parts of the output space: The A–S prices associated with a given output vector are simply found as the weighted sum of gradients of the linear facets of the estimated cost function along a radial contraction path of the observed output vector, where the weights are proportional to the length of the projected line segments. For every data point this can be computed using parametric linear programing.

However, for certain data points, and in particular for the observed productions that help span the empirical cost function, the cost function will in most cases be piecewise continuous differentiable along the radial contraction path and hence there may be multiple A–S prices for the same observation caused by lack of continuous differentiability on subintervals along this path. For reasons of transparency and simplicity, we suggest to overcome this problem by using a lexicographic goal programing approach with a predefined ordering of outputs to determine which outputs should be allocated most costs. Such orderings may, for instance,

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[1] For a recent general DEA reference, see e.g., Bogetoft and Otto (2010).

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be the result of a managerial prioritization and will provide unique A–S prices for all observations.[2]

Our approach, however, does not exclude the possibility of having zero A–S prices for some units (when these are referring to exterior facets). To solve this problem (as well as excluding the possibility of infinite A–S prices), one can use the "extended facet" approach in Olesen and Petersen (1996, 2003). This ensures well defined rates of substitution on the boundary of the convex envelopment of the data points, but in general this approach lacks operationability.

When it comes to inefficient production it seems that no previous papers have considered the consequences in relation to A–S pricing. Yet, countless empirical studies have shown that observed production data are often associated with considerable levels of technical inefficiency, see e.g., Bogetoft and Otto (2010).

To deal with inefficient production in the context of A–S cost allocation, we propose to invoke the *rational inefficiency paradigm* introduced in Bogetoft and Hougaard (2003) and further analyzed in Asmild, Bogetoft, and Hougaard (2009, 2013). This allows us to formulate specific assumptions concerning the behavior of inefficient firms, which in turn enables us to associate an efficient production with each inefficient observation in the sample. It is worth emphasizing that, as such, our suggested approach and associated results are independent of the way we estimate the cost function (although we are using non-parametric estimation for our empirical illustration).

In particular, firms can introduce inefficiency on either the cost (input) side or the production (output) side. Considering cost inefficiency we assume that the inefficient firm has revealed a constant fraction of overspending by its actual production choice. Thus, A–S prices connected with the cost efficient production can be scaled up with a radial cost efficiency index in order to obtain full cost allocation. We show that this approach is tantamount to viewing inefficiency as a fixed cost and to sharing this fixed cost in proportion to the A–S prices.

Looking at the output side we assume that firms introduce inefficiency by consuming outputs directly on the job. For example, some units of a given output may be produced in inferior quality and we can regard this as a kind of internal "consumption", which should not distort the estimation of A–S prices. A rationally inefficient firm would choose its actual (unobserved) production so as to maximize potential revenue given output prices and its observed cost. When we observe the actual output level lower than that it is because the firm has consumed the difference (slack) itself. We shall therefore argue that it is the allocatively efficient output combination that carries the cost and allocate costs accordingly using the A–S prices related to the allocatively efficient production.

We illustrate our approach using a data set concerning Danish waterworks. We use the same 2011 data that the regulator, the Water Division of the Danish Competition and Consumer Authority, used in their first regulatory cost benchmarking model, and we show how cost shares can be computed using our suggested A–S approach in case of a non-parametric estimation of the cost function.

The rest of the paper is organized as follows: Section 2 defines the standard Aumann–Shapley cost allocation rule for continuously differentiable cost functions. Section 3 introduces the convex envelopment approach to the estimation of the empirical cost function. We discuss how to calculate A–S prices from the estimated cost function and suggest how to deal with the lack of well defined A–S prices for all production units in Section 4. Section 5 deals with inefficient production in the context of A–S cost allocation building on the rational inefficiency paradigm. The illustrative application to data on Danish waterworks is presented in Section 6, and Section 7 contains final remarks.

[2] A far less operational approach would be to determine all facets involved (for data point in question) and define the associated A–S price as the (weighted) average of the gradients of these facets.

2. Aumann–Shapley cost allocation

Consider a joint production process resulting in n different outputs. Let $q \in \mathbf{R}^n_+$ be the (non-negative) output vector where q_i is the level of output i. The cost of producing any vector q is given by a non-decreasing cost function $C : \mathbf{R}^n_+ \to \mathbf{R}$. Initially, we assume that $C(0) = 0$, i.e., there are no fixed costs.

Let (q, C) denote a cost allocation problem and let ϕ be a cost allocation rule. The cost allocation rule specifies a unique vector of cost shares $x = (x_1, \dots, x_n) = \phi(q, C)$ for each output vector q and cost function C. The cost shares satisfy budget-balance, i.e.

$$\sum_{i=1}^{n} x_i = C(q)$$

where x_i is the cost share allocated to output i.

In particular, consider the class of continuously differentiable cost functions and let $\partial_i C(q) = \partial C(q)/\partial q_i$ be the partial derivative of C at q with respect to the ith argument.

Following Aumann and Shapley (1974), we define the Aumann–Shapley rule (A–S-rule) ϕ^{AS} as

$$\phi_i^{AS}(q, C) = \int_0^{q_i} \partial_i C\left(\frac{t}{q_i} q\right) dt = q_i \int_0^1 \partial_i C(tq) dt \quad \text{for all } i = 1, \dots, n. \tag{1}$$

It can be shown that this allocation is budget balanced, i.e., $\sum_{i \in N} \phi_i^{AS}(q, C) = C(q)$.

Also,

$$p_i^{AS} = \int_0^1 \partial_i C(tq) dt$$

can be seen as the unit cost of output i. This is known as the Aumann–Shapley price (A–S price) of output i. As such, the A–S cost shares, x_i^{AS}, are given by

$$x_i^{AS} = p_i^{AS} q_i \tag{2}$$

for all outputs $i = 1, \dots, n$.

The A–S rule can be seen as one (of several) possible extensions of average cost sharing to the multiple product case, see e.g. Hougaard (2009). Axiomatic characterizations are provided (independently) in Billera and Heath (1982) and Mirman and Tauman (1982). Following the latter, we here shortly recall the axioms characterizing A–S pricing $p^{AS}(C, q)$:

- (Rescaling) For some rescaling $q \mapsto \bar{q} = (\lambda_1 q_1, \dots, \lambda_n q_n)$, let $G(q) = C(\bar{q})$. Then, for all $i = 1, \dots, n$, $p_i(G, q) = \lambda_i p_i(C, \bar{q})$.
- (Consistency) Let $C(q) = G(\sum_{i=1}^n q_i)$. Then, for all $i = 1, \dots, n$, $p_i(C, q) = p_i(G, \sum_{i=1}^n q_i)$.
- (Additivity) Let $C(q) = G(q) + H(q)$. Then $p(C, q) = p(G, q) + p(H, q)$.
- (Positivity) Let C be non-decreasing at each $q' \leq q$. Then $p(C, q) \geq 0$.

Early examples of application can be found in Billera, Heath, and Raanan (1978) and Samet et al. (1984). More recent applications include Castano-Pardo and Garcia (1995), Haviv (2001), Tsanakas and Barnett (2003) and Bjørndal, Jörnsten, Koster, and Delfman (2005).

Example 1. Consider the simple case where the cost function is homogeneous of degree k, i.e., $C(tq) = t^k C(q)$ for $t \in [0, 1]$. Here it is clear that for all $i \in N$, the A–S prices become

$$p_i^{AS} = \partial_i C(q) \int_0^1 t^{k-1} dt = p_i^{MC} \frac{1}{k},$$