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Interfaces with Other Disciplines

The weighted additive distance function

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ABSTRACT

Distance functions in production theory are mathematical structures that characterize the belonging to the reference technology through a numerical value, behave as technical efficiency measures when the focus is analyzing an observed input–output vector within its production possibility set and present a dual relationship with some support function (profit, revenue, cost function). In this paper, we endow the well-known weighted additive models in Data Envelopment Analysis with a distance function structure, introducing the Weighted Additive Distance Function and showing its main properties.

1. Introduction

Efficiency evaluation in production of whatever type of firm and public organization has been a relevant topic for managers and policy makers as well as an area of interest from a practical and methodological point of view in both operations research (OR) and economics. The main aim of such assessment is to analyze the efficiency of a so-called DMU (Decision Making Unit), which uses several inputs to produce several outputs, by comparing its performance with respect to the boundary of a technology using to that end a sample of other DMUs operating in a similar technological environment.

Chronologically speaking, the empirical estimation of the underlying technologies began in the area of economics with the application of regression analysis and Ordinary Least Squares (OLS) to estimate a parametrically specified 'average' production function, e.g., a Cobb–Douglas function (Cobb & Douglas, 1928). Later, Farrell (1957) was the first in showing, for a single output and multiple inputs, how to estimate an isoquant enveloping all the observations. Farrell's paper inspired other authors to continue this line of research estimating production functions that envelop all the observations of the sample by either a non-parametric piece-wise linear technology or a parametric function. The first possibility was taken up by Banker, Charnes, and Cooper (1984), Charnes, Cooper, and Rhodes (1978) and others, resulting in the development of DEA (Data Envelopment Analysis), adopted mostly by engineers and OR practitioners; while the latter approach was taken up by Aigner and Chu (1968), Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977) and others, subsequently resulting in the development of the deterministic and stochastic frontier models, adopted mostly by economists and statisticians.

In contrast to the parametric literature on efficiency, where the measurement of technical efficiency in the context of multiple-outputs is based on a few measures, fundamentally the Shephard input and output distance function and the directional distance function, the first years of life of DEA witnessed the introduction of many different technical efficiency measures, such as the Russell input and output measures of technical efficiency and their graph extension; the Russell Graph Measure of technical efficiency (see Färe, Grosskopf, & Lovell, 1985), the additive model (Charnes, Cooper, Golany, Seiford, & Stutz, 1985), the Range-Adjusted Measure (Cooper, Park, & Pastor, 1999) and the Enhanced Russell Graph (Pastor, Ruiz, & Sirvent, 1999) or Slacks-Based Measure (Tone, 2001), to name but a few. One of the reasons for the introduction of many different technical efficiency measures in DEA is the piece-wise linear nature of the boundary of the technology. In this context, a notion that comes into play is Pareto-efficiency (Koopmans, 1951). Pareto-efficiency, however, seems not be a problem for the parametric approach, where the functional forms utilized to model the frontier of production are usually smooth. On the contrary, it has been a recurring theme in DEA. In particular, the additive model by Charnes et al. (1985) was the first graph linear model[1] that ensured that the evaluated DMU was compared exclusively with respect to the set of Pareto-efficient points in the

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[1] The Russell Graph Measure (Färe et al., 1985) also projects the evaluated points onto the Pareto-efficient frontier but its objective function is not linear.

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input–output space. From this model, DEA researchers have introduced some modifications of the original additive model weighting the slacks that appear in the objective function (see, for example, Cooper et al., 1999, Cooper, Pastor, Borras, Aparicio, & Pastor, 2011a, Lovell & Pastor, 1995 and Pastor, Aparicio, Alcaraz, Vidal, & Pastor, 2015) in order to measure technical inefficiency using the strongly efficient frontier as a reference. This existence of a different battery of tools for estimating technical efficiency in the parametric and non-parametric world reveals the importance in DEA in measuring efficiency with respect to the Pareto-efficient frontier.

As another matter of fact, most of the classical results and applications in microeconomics related to the measurement and decomposition of overall efficiency, in terms of technical and allocative (price) efficiency, and the estimation of productivity change over time from panel data are based on the notion of distance function and duality theory. A distance function behaves, in fact, as a technical efficiency measure when an observation belonging to the reference technology is evaluated, with a meaning of 'distance' from the assessed interior point to the boundary of the production possibility set.[2] Also, the distance functions have dual relationships with well-known support functions in microeconomics, as the profit function or the cost and revenue functions. Another interesting feature of these functions is that they characterize the belonging or not belonging to the technology by means of a sign, as happens with the directional distance function, or by being greater or lesser than one, as in the case of the Shephard distance functions. This feature easily allows measuring productivity change over time; even in the case of cross-period evaluation when a unit observed in period t+1 under assessment is outside the technology corresponding to period t.

In a non-parametric framework, except for the case of resorting to typical parametric tools, i.e. the Shephard distance functions and the directional distance function, whose duality relationships with classical support functions were proved for production possibility sets fulfilling general axioms (e.g. convexity) and, therefore, may be applied to polyhedral technologies, most attempts at estimating overall efficiency and productivity change in DEA neglect the notion of distance function, a fact that contrasts significantly with the traditional view of economics of production, where both this concept and duality are the cornerstones of the applied theory. In this way, some researchers have tried to use additive-type models in DEA for measuring not only technical inefficiency but also productivity and profit inefficiency without resorting directly to the notion of distance function. To that end, they have somehow adapted a 'pure' technical inefficiency measure, the additive model, to be used in other contexts (e.g. productivity), exploiting features that are not specific to a technical efficiency measure but rather a distance function.

Regarding productivity, Grifell-Tatjé, Lovell, and Pastor (1998) introduced the quasi-Malmquist productivity index as a modification of the traditional Malmquist index based on an output-oriented weighted additive model. Additionally, Du, Chen, Chen, Cook, and Zhu (2012), Premachandra, Chen, and Watson (2011) and Du, Wang, Chen, Chou, and Zhu (2014) use an additive-type approach to check superefficiency *a la* Andersen and Petersen (1993). However, their model always provides non-negative values at the optimum whereas we seek to distinguish the belonging or not belonging to the reference technology by the sign of the optimal value of the additive model (non-negative/negative). Other related literature is one that addresses the issue of measuring and decomposing input-specific productivity. As for this literature, the different approaches that can be found to measure and decompose input-specific productivity change over time resort to a version of the input-oriented weighted additive model, although the different authors invoke the name of other measures as the Russell input measure (Oude Lansink & Ondersteijn, 2006), the input-oriented version of the directional slacks-based measure of inefficiency by Mahlberg and Sahoo (2011), based on Fukuyama and Weber (2009), or the Färe and Grosskopf (2010) slacks-based measure of efficiency in the directional input distance function context (Chang, Hu, Chou, & Sun, 2012, Skevas & Oude Lansink, 2014 and Kapelko, Horta, Camanho & Oude Lansink, 2015). In these cases, the corresponding slacks are constrained to be non-positive for cross-period evaluation by previously determining whether the assessed unit belongs to the reference technology or not and, consequently, the measures obtained are negative for units located out of the production possibility set and non-negative for units placed inside this set, a feature more usual of a distance function than a technical efficiency measure.

As for the measurement of overall efficiency, as far as we are aware, there have only been two attempts at estimating and decomposing profit inefficiency through additive-type models in DEA. The first one is that based on the paper of Cooper et al. (1999). These authors focus their interest on the traditional difference-form to measure profit inefficiency, i.e. optimal profit minus actual profit. However, this approach is homogeneous of degree one in prices and, additionally, the value of the technical component is not independent of alternative optimal solutions of the additive model. For these reasons, Cooper, Pastor, Aparicio, and Borras (2011b) took up where Cooper et al. (1999) left off and proposed a normalized profit inefficiency measure, which can be decomposed into technical and allocative inefficiencies by means of the optimal value of the weighted additive model.

Given the above discussion, there are two main objectives we pursue in this paper. First, since researchers are using additive-type models of technical efficiency for estimating profit inefficiency and productivity change, this paper is interested to endow the weighted additive model in DEA with a distance function structure. Second, this paper particularly shows that the traditional profit function is dually linked with the weighted additive model. In this respect, it is worth mentioning that so far only the dual correspondences of the profit function with the directional distance function (Chambers, Chung, & Färe, 1998) and with the Hölder distance function (Briec & Lesourd, 1999) were known. Consequently, we establish a new dual correspondence limited to the DEA framework. To achieve both these objectives, we introduce the weighted additive distance function and resort to duality theory. The new approach proposed in this paper can also be useful, from the point of view of practice, to managers and policy makers in their decision making. Weighted additive models have been utilized by practitioners in order to determine technical inefficiency considering the Pareto-Koopmans definition of efficiency. Endowing the weighted additive model with a structure of distance function will allow practitioners to use the same measure in situations where market prices are available (overall inefficiency) or a panel data is accessible (productivity change), opening the field of application of the original weighted additive models in a natural way.

The remainder of the paper is organized as follows: In Section 2, we briefly detail the definition and properties of the weighted additive models in DEA. We introduce the definition and main properties of the weighted additive distance function in Section 3. In Section 4, the dual relationship between the weighted additive distance function and the profit function in DEA is shown. In Section 5, we present the conclusions.

[2] In this paper, we distinguish two situations. The first one is related to any distance function when it is utilized for evaluating a set of DMUs with respect to their contemporaneous production technology. In this case, hereafter, we will speak of 'technical efficiency'. Otherwise, we will simply speak of the distance from the assessed point to the frontier of the reference production possibility set. This second scenario can occur, for example, if the evaluated unit is observed in period t of time and the reference technology is estimated from observations of period t+1.