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Lower bounding procedure for the asymmetric quadratic traveling salesman problem

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ABSTRACT

In this paper we consider the Asymmetric Quadratic Traveling Salesman Problem (AQTSP). Given a directed graph and a function that maps every pair of consecutive arcs to a cost, the problem consists in finding a cycle that visits every vertex exactly once and such that the sum of the costs is minimal. We propose an extended Linear Programming formulation that has a variable for each cycle in the graph. Since the number of cycles is exponential in the graph size, we propose a column generation approach. Moreover, we apply a particular reformulation-linearization technique on a compact representation of the problem, and compute lower bounds based on Lagrangian relaxation. We compare our new bounds with those obtained by some linearization models proposed in the literature. Computational results on some set of benchmarks used in the literature show that our lower bounding procedures are very promising.

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1. Introduction

The Traveling Salesman Problem (TSP) is one of the most studied optimization problems. Given a graph $G = (V, A)$ with costs c_e, $e \in A$, the problem consists in finding a cycle C that visits each vertex in V exactly once (i.e., a Hamiltonian cycle), and such that the total cycle cost is minimal. In its most common form, the TSP has a linear cost function, equal to the sum of the costs c_e of each arc in C. Now, consider a variant of the TSP having a non-negative interaction cost Q_{ef} for every pair of consecutive arcs $e, f \in A$. Because the cost of a cycle can be computed as a quadratic function, this problem is called Quadratic Traveling Salesman Problem (QTSP) and is NP-hard as shown in Fischer et al. (2014). As for the linear counterpart, the QTSP has a symmetric and an asymmetric variant. In this paper we focus on the Asymmetric Quadratic Traveling Salesman Problem (AQTSP) which appears to be more challenging. A preliminary version of the results presented here together with the study of the Symmetric Quadratic Traveling Salesman Problem (SQTSP) have been published as conference proceedings in Rostami, Malucelli, Belotti, and Gualandi (2013).

The QTSP was introduced as an application of bioinformatics by Jäger and Molitor (2008) but it can also be applied in robotics and telecommunications. In robotics, a variant of QTSP called *Angle-TSP* can be used for the optimization of robot paths in terms of energy consumption. The Angle-TSP problem seeks to minimize the total angle of a TSP tour for a set of points in the Euclidean space where the angle of a tour is the sum of the direction changes at all points (Aggarwal, Coppersmith, Khanna, Motwani, & Schieber, 2000). Also, the QTSP can be viewed as a generalization of the Reload Cost TSP (RTSP) introduced by Amaldi, Galbiati, and Maffioli (2011). In the RTSP, one is given a graph whose each arc is assigned a color and there is a *reload* cost when passing through a node on two arcs that have different colors.

The QTSP has been introduced quite recently and its literature is rather limited. In particular, the QTSP has been tackled by Fischer et al. (2014) with heuristic algorithms based on well-known heuristics for the TSP, with an ad-hoc branch-and-bound, and with a branch-and-cut approach based on a linearization of a 0-1 Quadratic Programming formulation of the problem. In Fischer and Helmberg (2013) and Fischer (2014) used the polyhedral structure of linearized integer programming formulations to develop a branch-and-cut algorithm for the SQTSP and AQTSP, respectively, that are the current state-of-the-art for QTSP.

In this paper we study lower bounding procedures for the AQTSP. Considering the special structure of the quadratic costs of the AQTSP, we present an extended Linear Programming formulation of the problem with an exponential number of variables that is solved via Column Generation (CG). The basic idea is to

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have a variable for each cycle of G. This yields a pricing subproblem that consists in finding a cycle of minimum quadratic cost. We formulate the pricing subproblem as a 0–1 Quadratic Program, which is linearized and solved with standard techniques. We resort to stabilization techniques to overcome the tailing-off effect of the CG approach. We also present the linearized integer programming formulation proposed by Fischer (2014). This formulation is, in fact, based on the standard Asymmetric Traveling Salesman Problem (ATSP) formulation of Dantzig, Fulkerson, and Johnson (1954) which has an exponential number of constraints. In order to avoid this complication we modified the proposed model by using the compact formulation of Miller, Tucker, and Zemlin (1960) with a polynomial number of variables and constraints. To generate a tighter representation of the problem, we develop a Mixed Integer Linear Programming (MILP) formulation that make use of a particular application of the Reformulation-linearization technique (RLT) (Adams & Sherali, 1986; 1990). We develop an effective Lagrangian relaxation scheme to compute the associated lower bound.

2. Problem statement and linearizations

Consider a complete directed graph $G = (V, A)$ with vertex set $V = \{1, 2, \ldots, n\}$, arc set A, and cost function Q that maps every pair of consecutive arcs $e = (i, j), f = (j, k) \in A$ to a non-negative integer cost $Q_{ef} = Q_{ijk}$. The AQTSP seeks a directed tour (i.e., a directed cycle passing through every vertex exactly once) of minimum cost. Let the binary variable x_{ij} be equal to 1 if the arc $(i, j) \in A$ belongs to the minimum cost tour, and zero otherwise. A formulation for the AQTSP is then given by

$$\text{AQTSP:}\quad \min \sum_{(i,j),(j,k)\in A:\ k\neq i} Q_{ijk} x_{ij} x_{jk}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij} = 1 \quad i \in V \tag{1}$$

$$\sum_{i:(i,j)\in A} x_{ij} = 1 \quad j \in V \tag{2}$$

$$\sum_{\substack{i\in S,\, j\notin S:\\ (i,j)\in A}} x_{ij} \geq 1 \quad S \subset V,\ 2 \leq |S| \leq n-2 \tag{3}$$

$$x_{ij} \in \{0, 1\} \quad (i, j) \in A. \tag{4}$$

where constraints (1) and (2) force to select a single outgoing arc and a single incoming arc for each node, respectively, and constraints (3) are the well known subtour elimination constraints (Dantzig et al., 1954). To ease the argumentation, we define the new set $\mathcal{A}$ as follows:

$$\mathcal{A} = \{(i, j, k) : (i, j) \in A, (j, k) \in A, i \neq k\}.$$

The AQTSP formulation presented here is based on the ATSP formulation of Dantzig et al. (1954) which has an exponential number of constraints. Since, further on, we will need a more compact formulation we adapt the MTZ formulation of Miller et al. (1960) to the quadratic case. That formulation fixes node 1 as a "depot", which the salesman must leave at the start of the tour and return to at the end of the tour. Let u_j be a continuous variable representing the position of node j in the tour. Then the MTZ type formulation for the AQTSP is given by

$$\text{AQTSP}_{\text{MTZ}}:\quad \min \sum_{(i,j,k)\in\mathcal{A}} Q_{ijk} x_{ij} x_{jk}$$

$$\text{s.t.}\quad u_i - u_j + (n-1)x_{ij} \leqslant n-2 \quad i, j \geqslant 2;\ i \neq j \tag{5}$$

$$1 \leqslant u_j \leqslant n-1 \quad j \geqslant 2$$
$$(1), (2), (4). \tag{6}$$

where constraints (5) ensure that, if the salesman travels from node i to node j, then the position of node j is one more than that of node i, and constraints (5) together with the bounds (6) ensure that each non-depot node is in a unique position.

2.1. Linearization

The main difficulty of the AQTSP and AQTSP$_{\text{MTZ}}$, in addition to the TSP problem structure, lies in the quadratic structure of the objective function. One of the most natural ways for solving the problem is to linearize the quadratic terms $x_{ij}x_{jk}$ for all $(i, j, k) \in \mathcal{A}$ using the standard linearization technique of Glover and Woolsey (1974). The approach is based on the introduction of new non-negative binary variables $y_{ijk} = x_{ij}x_{jk}$ which satisfy the following set of constraints:

$$y_{ijk} \leq x_{ij}, \quad y_{ijk} \leq x_{jk}, \quad \text{and} \quad y_{ijk} \geq x_{ij} + x_{jk} - 1.$$

The main disadvantage of the standard linearization is that it does not take the problem structure into account. Taking into account the problem structure to connect the x and y variables, Fischer (2014) introduced a mixed integer linear programming with continuous variables y_{ijk} such that

$$y_{ijk} = x_{ij}x_{jk} \quad \text{for all} \quad (i, j, k) \in \mathcal{A}.$$

The formulation has the following form:

$$\text{LPF:}\quad \min \sum_{(i,j,k)\in\mathcal{A}} Q_{ijk} y_{ijk}$$

$$\text{s.t.} \sum_{k:(i,j,k)\in\mathcal{A}} y_{ijk} = x_{ij} \quad (i, j) \in A \tag{7}$$

$$\sum_{k:(k,i,j)\in\mathcal{A}} y_{kij} = x_{ij} \quad (i, j) \in A \tag{8}$$

$$y_{ijk} \geq 0 \quad (i, j, k) \in \mathcal{A}$$
$$(1) - (4). \tag{9}$$

where constraints (7) and (8) enforce $y_{ijk} = x_{ij}x_{jk}$.

Using the AQTSP$_{\text{MTZ}}$ compact formulation discussed above, we define a compact formulation of LPF, called LPF$_{\text{MTZ}}$ as follows:

$$\text{LPF}_{\text{MTZ}}:\quad \min \left\{ \sum_{(i,j,k)\in\mathcal{A}} Q_{ijk} y_{ijk} \ : (1), (2), (4) - (9) \right\}.$$

Padberg and Sung (1991) showed that the LP relaxation of the MTZ formulation yields an extremely weak lower bound for the ATSP. However, our computational results show that this is not always true for the AQTSP.

3. RLT applied to the AQTSP

To obtain an MILP formulation for the AQTSP, we apply a scheme based on the Reformulation-linearization technique (RLT). As far as the tightness of the bounds is concerned, the RLT base approaches are among the most successful lower bounding approaches for many combinatorial optimization problems (Adams, Guignard, Hahn, & Hightower, 2007; Adams & Johnson, 1994; Rostami & Malucelli, 2015; Sherali & Driscoll, 2002). The full application of the RLT for general zero-one polynomial programs (Adams & Sherali, 1986; 1990), transforms the original problem into an MILP via two basic steps of *reformulation* and *linearization*. In the reformulation step, the constraints are multiplied by the binary variables and their complements to construct redundant nonlinear constraints. In the linearization step, the objective and constraints

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