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Shared resource capacity expansion decisions for multiple products with quantity discounts

Jonathan E. Jackson[a], Charles L. Munson[b,*]

[a] School of Business, Providence College, Providence, RI 02918, USA
[b] Carson College of Business, Washington State University, Pullman, WA 99164-4746, USA

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ABSTRACT

When multiple products compete for the same storage space, their optimal individual lot sizes may need to be reduced to accommodate the storage needs of other products. This challenge is exacerbated with the presence of quantity discounts, which tend to entice larger lot sizes. Under such circumstances, firms may wish to consider storage capacity expansion as an option to take full advantage of quantity discounts. This paper aims to simultaneously determine the optimal storage capacity level along with individual lot sizes for multiple products being offered quantity discounts (either all-units discounts, incremental discounts, or a mixture of both). By utilizing Lagrangian techniques along with a piecewise-linear approximation for capacity cost, our algorithms can generate precise solutions regardless of the functional form of capacity cost (i.e., concave or convex). The algorithms can incorporate simultaneous lot-sizing decisions for thousands of products in a reasonable solution time. We utilize numerical examples and sensitivity analysis to understand the key factors that influence the capacity expansion decision and the performance of the algorithms. The primary characteristic that influences the capacity expansion decision is the size of the quantity discount offered, but variability in demand and capacity per unit influence the expansion decision as well. Furthermore, we discover that all-units quantity discounts are more likely to lead to capacity expansion compared to incremental quantity discounts. Our analysis illuminates the potential for significant savings available to companies willing to explore the option of increasing storage capacity to take advantage of quantity discount offerings for their purchased products.

1. Introduction

With ever-shrinking margins in today's marketplace, proper inventory management has become more vital than ever before. Purchasing managers typically procure many products simultaneously, often subject to an inventory-based resource constraint (e.g., warehouse space). Basic inventory models, such as the economic order quantity (EOQ) model, assume that procurement decisions relate to a single product with no constraints on the order size. These basic models are well-known and easy to solve analytically. However, the addition of a common resource constraint, multiple products, and quantity discounts significantly complicates the determination of optimal order quantities.

Inventory-based common resource constraints typically come in one of two forms: warehouse constraints or financial constraints. A warehouse can be constrained in terms of volume (or square footage), weight, or number of units. The most common warehouse constraint is square footage, as the drive toward "lean" in many industries today severely limits available warehouse space. Financial constraints might arise, for example, from a maximum inventory value that insurance covers or from a credit line available to purchase inventory. A natural, but often forgotten, question is, "Would it be valuable for our firm to increase the capacity of our common resource?" Too often managers may simply assume that capacity is fixed, but in reality, more warehouse space may be available for purchase, or unused warehouse space may be convertible for lease.

Any expansion decision comes with a cost, so what is the benefit? The vast majority of businesses have opportunities to receive quantity discounts for at least some of their purchased products (Munson & Rosenblatt, 1998). By increasing the resource capacity, firms open up more opportunities to take advantage of quantity discounts. The model in this paper accommodates both of the common quantity discount forms: *all-units* and *incremental* (Hadley & Whitin, 1963). Specifically, we address the following research question: When a firm faces all-units or incremental quantity discount schedules for multiple products in a common resource-constrained inventory system, how much capacity should exist, and how many

* Corresponding author. Tel.: +1 509 335 3076; fax: +1 509 335 3857.
E-mail addresses: jjackso9@providence.edu (J.E. Jackson), munson@wsu.edu (C.L. Munson).

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units of each item should the firm order given that capacity limitation?

This paper provides purchasing managers with solution algorithms to answer these questions. Specifically, the algorithms simultaneously determine best order quantities for each product along with the optimal common resource capacity level when the firm receives all-units and/or incremental quantity discounts from its suppliers. While previous research has examined the multiple-product lot-sizing decision with a fixed capacity, to the best of our knowledge this study is the first that treats capacity as a decision variable in the presence of quantity discounts. Furthermore, while previous literature restricts the functional form of capacity cost (e.g., convex or linear), our model accommodates any functional form (e.g., concave, convex–concave, concave-convex, etc.), which increases applicability by accurately modeling the many different ways in which capacity expansion may create costs. Through the numerical studies developed in Section 6, it becomes clear that the introduction of the common resource capacity as a decision variable presents opportunities for considerable savings from quantity discounts that may outweigh the corresponding increase in cost associated with capacity expansion. This especially holds true in the presence of all-units quantity discounts compared to incremental quantity discounts. We find that the size of the quantity discount offered, along with variability in demand and capacity per unit, have the most influence on the capacity expansion decision. The modeling concept of treating capacity expansion as a decision variable opens up directions for future research involving quantity discounts and joint replenishment inventory problems.

The remainder of this paper is organized as follows. In Section 2, we review relevant literature on quantity discounts and the capacitated common resource problem. In Section 3, we introduce the baseline model and algorithm when facing all-units quantity discounts. This is followed in Section 4 by an extension of the baseline model to incorporate capacity *reduction* decisions (leading to potential capacity cost savings) along with capacity expansion decisions. The model and solution algorithm are then modified to handle incremental quantity discounts in Section 5. In Section 6, we describe numerical studies that test the performance of the algorithms along with sensitivity analysis that identifies the key parameters influencing the capacity expansion decision. Finally, in Section 7, we conclude the study and identify directions for future research.

2. Literature review

This paper draws from two expansive areas of the operations management literature: quantity discounts and the capacitated common resource problem. Our literature review critically analyzes multiple facets of these areas of literature to identify research gaps and motivate the need for our algorithms to assist in this complex decision of simultaneously making hundreds or thousands of order quantity decisions while also considering a potential capacity expansion or reduction. Within the capacitated common resource problem literature, there are three characteristics of interest: pricing structure, ordering structure, and capacity flexibility. We describe how researchers have treated these characteristics. From there, we highlight critical areas that are currently understudied.

2.1. Quantity discounts

Several hundred academic articles on quantity discounts have appeared. Benton and Park (1996) as well as Munson and Rosenblatt (1998) summarize the work published through the turn of the century and provide an overview of the landscape of the quantity discount literature. A steady stream of quantity discount papers has continued since then (e.g., Hammani, Temponi, & Frein, 2014, Manerba & Mansisi, 2012, Munson & Hu, 2010, Rubin & Benton, 2003), including a recent handbook (Munson & Jackson, 2014) that provides practical motivations as well as a thorough review of the quantity discount literature. As mentioned previously, our models accommodate both all-units (Section 3) and incremental (Section 5) quantity discounts. Most introductory operations management textbooks (e.g., Heizer, Render, & Munson, 2017) provide the basics of all-units quantity discounts, while many intermediate textbooks (e.g., Chopra & Meindl, 2010) describe solution techniques for incremental quantity discounts.

2.2. Common resource capacity problems

Previously published models that address the common resource capacity problem have three key characteristics: pricing structure, ordering structure, and capacity flexibility. Table 1 provides a summary of prior literature and the characteristics of each article. There are two common pricing structures: fixed pricing (i.e., no quantity discounts) and variable pricing with quantity discounts. Hadley and Whitin (1963) and Johnson and Montgomery (1974) were among the first to analyze the fixed pricing (undiscounted) problem. They use an ordering structure whereby each product has an independent cycle length, i.e., time between orders. This type of ordering structure forces the firm to prepare for the worst-case scenario when all products are ordered at once and arrive simultaneously. Lagrangian relaxation is the most popular solution technique to solve the undiscounted common resource problem with independent cycle times. Rosenblatt (1981) and Rosenblatt and Rothblum (1990) modify the ordering structure to have every product on a fixed cycle length. The replenishment points are then phased within the fixed cycle length. This technique lowers the maximum inventory level by eliminating the possibility of the worst-case scenario that independent cycle lengths might produce.

There are several papers dedicated to solving the capacitated common resource problem with quantity discounts, but all of them assume that capacity is fixed. Pirkul and Aras (1985) wrote the seminal paper analyzing all-units quantity discounts, which solves the problem with each product having an independent

Table 1
Summary of prior literature.

Reference	Pricing structure[a]	Ordering structure[b]	Capacity flexibility[c]
Güder et al. (1994)	I	I	F
Güder and Zydiak (1997)	A	NS	F
Güder and Zydiak (2000)	A	F	F
Hadley and Whitin (1963)	F	I	F
Haksever and Moussourakis (2005)	F	F,I	F
Haksever and Moussourakis (2008)	I	F	F
Hall (1988)	F	F	V
Johnson and Montgomery (1974)	F	I	F
Minner and Silver (2007)	F	I	F
Moussourakis and Haksever (2008)	A	F,I	F
Pirkul and Aras (1985)	A	I	F
Rosenblatt (1981)	F	F,I	F
Rosenblatt and Rothblum (1990)	F	F	V
Rubin and Benton (1993)	A	I	F
Rubin and Benton (2003)	I	I	F
Zhang (2010)	F	NV	F

[a] A = all-units, F = fixed, I = incremental.
[b] F = fixed, I = independent, NS = non-stationary, NV = newsvendor.
[c] F = fixed, V = variable.