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Democratic elections and centralized decisions: Condorcet and Approval Voting compared with Median and Coverage locations

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ABSTRACT

In this study, we focus on the quality of Condorcet and Approval Voting winners using Median and Maximum Coverage problems as benchmarks. We assess the quality of solutions by democratic processes assuming many dimensions for evaluating candidates. We use different norms to map multidimensional preferences into single values. We perform extensive numerical experiments. The Condorcet winner, when he/she exists, may have very high quality measured by the Median objective function, but poor quality measured by the Maximum Coverage problem. We show that the Approval Voting winner is optimal when the quality is measured by the Maximum Coverage objective and fairs well when the Median objective is employed. The analyses further indicate that the number of voters and the distance norm may increase, while the number of candidates and dimensions may decrease the quality of democratic methods.

1. Introduction

It has been generally acknowledged since Arrow's (Arrow, 2012) work in the 1950s that voting methods have severe limitations. Elections may result in paradoxes such as winners not representing voters' top preferences, or even no outright winners. These paradoxes may occur with different methods, and either increase or decrease with the number of voters (Balinski & Laraki, 2014; Plassmann & Tideman, 2014).

Voting methods are defined by rules indicating how voters' preferences are elicited and ordered to identify winners (De Sinopoli, 2000). In the Downsian model, a voter's preference for a candidate increases with their proximity in a multidimensional space representing different utility criteria (Gouret, Hollard, & Rossignol, 2011; Henry & Mourifié, 2013; Merrill, 1985). According to Bhadury, Griffin, Griffin, and Narasimhan (1998), those distances are weighted according to the importance given by voters to each dimension. The ideal voting method would elicit a clear winner that maximized the products of voters' preferences and weights along each utility dimension.

In the Condorcet method, candidates are compared pairwise (Taylor & Pacelli, 2008). Each voter returns one ballot with a ranked order of preferred candidates. The candidate with more votes in the pairwise competition beats the other candidate. A candidate who beats all other candidates is proclaimed the winner. If there are ties, then we consider the winner the candidate who beats all others after ties are removed or, equivalently, the candidate who does not lose to any other candidate. If even after removing ties more than one candidate meets the criterion, then the result is decided randomly. It is possible not to have a winner due to what is known as the Condorcet Paradox (Balinski & Laraki, 2014; Plassmann & Tideman, 2014; Taylor & Pacelli, 2008).

In Approval Voting, each voter makes an unranked list of candidates they approve. The candidates with the most approvals win. If ties occur, they are broken randomly. Brams and Fishburn (1978, 2007); Rapoport and Felsenthal (1990); Taylor and Pacelli (2008).

Election planners need to identify methods that increase the odds of results being paradox-free and maximizing utility to voters in their particular context. The literature offers a broad choice of election methods and scoring systems (Balinski & Laraki, 2014; Plassmann & Tideman, 2014). Moreover, the quality of a method (i.e. how its results compare to an optimal, centralized solution by an "omniscient and benevolent dictator" (Mueller, Philpotts, & Vanek, 1972, p. 66)) may vary depending on the number of candidates and voters (Arrow, 2012; Drezner & Menezes, 2015; Plassmann & Tideman, 2014; Pritchard & Slinko, 2006).

Given the well-known failures of voting methods (Balinski & Laraki, 2014), several studies have compared their relative

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performance when applied to different contexts. Klamler and Pferschy (2007) compare voting preferences for local versus global "tours" using Borda, Plurality, Simple Majority rules, and Approval Voting, with mixed results. Buenrostro, Dhillon, and Vida (2013) investigate necessary levels of agreement between voters using Borda rule, Plurality rule, Approval Voting, and Relative Utilitarianism leading to dominance-solvable games. They suggest that Approval Voting yields the best results. Merrill (1985) performs a simulation study of six methods to estimate the fraction of outcomes coinciding with the Condorcet winner. His statistical analysis uses both ℓ_1 and ℓ_2 norms.

Campos-Rodríguez and Moreno-Pérez (2008) present an algorithm to find Condorcet solutions when the solution is not a singleton but a set of given cardinality. Menezes and Huang (2015) study the theoretical quality of a Condorcet solution as a function of the solution set cardinality. They find that the relative efficiency of a Condorcet solution is $\sqrt{2}$ in the worst case, and approaches 1 as the number of alternatives increases. They use a discrete model and the Euclidean distance (ℓ_2 norm) as proxy for dis-utility.

The Median problem is either defined in a network (Hakimi, 1964), or termed the Weber problem in continuous space (Drezner, Klamroth, Schöbel, & Wesolowsky, 2002). Hansen and Thisse (1981) use a network setting to derive upper bounds on the ratio of the Weber objective function at the Condorcet solution and the optimal solution. They prove that this ratio is bounded by 3. Further properties of Condorcet network problems based on graph-theoretic results are presented by Hansen, Thisse, and Wendel (1986), but mostly focusing on local rather than global solutions. Hansen and Labbè (1988) present polynomial algorithms to determine the sets of Condorcet and Simpson points of a network. Noltemeier, Spoerhase, and Wirth (2007) investigate the special topology of tree networks. For the case of single voting location problems on trees, algorithms are developed for Condorcet and Simpson cases. Wendel and Thorson (1974) prove that when the problem uses either the ℓ_1 or the ℓ_∞ norm, the Condorcet solution point is the Weber solution point.

There are similarities between the models herein and competitive location models. Customers "vote" for the best retail facility by patronizing it. There are two main branches of thought in facility location studies depending on assumptions about consumer behavior. One follows the gravity model suggested by Reilly (1931) and applied to a retail competitive environment by Huff (1964, 1966). Locating a facility by this model is first introduced by Drezner (1994b). In the other approach, a customer patronizes the closest facility (Drezner, 1982; Hakimi, 1983; 1986; Hotelling, 1929). Drezner (1994a) suggests that the facility with highest utility may not be the closest one. For a review of competitive facility location the reader is referred to Berman, Drezner, Drezner, and Krass (2009).

Campos-Rodríguez and Moreno-Pérez (2000) present a variation of the classic approach to Condorcet problems in location analysis. They introduce an insensitivity factor for voters when the difference between distances to two locations is less than α. This approach is reasonable because the decision by the voter is not clear when distances are similar. Another way to model those situations is by a stochastic approach. Since every demand point consists of many voters, their votes are shared across different locations based on their relative distances to the demand point. This approach is similar to the gravity model in competitive facility location (Drezner, 1994a; Huff, 1964; 1966; Reilly, 1931).

Drezner and Menezes (2015) perform extensive numerical computations to assess the quality of Condorcet solutions compared with the Weber problem in two dimensions.

In this paper, we compare Condorcet and Approval Voting with traditional approaches to centralized decision making, namely the median and maximum coverage models. There is a distance between each candidate and a node of voters on each of r dimensions. The shorter the distance between the candidate and a node, the higher the utility of the candidate to the voter, making distance a proxy for dis-utility (Gouret et al., 2011). We use an ℓ_p norm for computing the distance from the candidate to the node taking into account all utility dimensions. As far as we know, no studies have assessed the quality of Approval Voting and Condorcet methods in relation to the Maximum Coverage problem. Also, studies on the effects of different norms on the quality of a method are not yet available. Most studies are analyzed in $\mathbb{R}$ or $\mathbb{R}^2$ - the Euclidean space (Henry & Mourifié, 2013; Merrill, 1985), but not in higher dimensions spaces.

2. Research framework

We compare the total utility of a democratic election winner with the optimal choice obtained through a centralized process. In particular, we compare the Condorcet and Approval Voting methods with the Median (sum of all utilities) and Maximum Coverage problem (the candidate within a fixed distance of the highest number of voters). The Maximum Coverage problem is also referred to as the "Coverage" problem.

The Coverage and Median solutions are obtained by a central planner (benevolent dictator) having complete information about voters' preferences. Quality is defined by the ratio between the values of the centralized objective at the democratic solution and at the optimal centralized solution. By definition, this ratio is equal to or greater than one. The closer this ratio is to one, the better is the quality of the democratic solution. We assess the quality of each of the two democratic methods with each of the two centralized methods.

We compare methods by varying four different election parameters. The first parameter is the number of nodes representing the electorate. Each node in a network represents a different number of voters having similar preferences over candidates. Second, given a stable set of strategies by voters in Approval Voting, their influence on an election may depend on the number of candidates (Fishburn & Brams, 1981; Rapoport & Felsenthal, 1990). Thus, we evaluate the relative quality of voting methods based on the number of candidates. The third parameter is the norm used to estimate distance between candidates and voters. Since our primary objective is to measure the utility of a candidate to a voter, we want to explore different magnitudes on those utilities. There is abundant literature on function spaces and, among them, a family of norms defined by ℓ_p norms (Tao, 2008). The fourth parameter is the number of dimensions on which voters assess candidates (Merrill, 1985). For example, reviewing applicants to a faculty position might be based on a few or many criteria from a list such as teaching experience, research publications, committee service, editorial boards, grants, etc.

In Approval Voting, we assume a critical distance so that voters approve candidates within that critical distance. To simulate Approval Voting, we apply different levels of the critical distance. Baron, Altman, and Kroll (2005) studied parochial voting and found that when the critical distance is short, only candidates with high utility to the voter or their group are approved, and when it is long, candidates with greater average utility to all voters may also be approved.

3. The models

In this section, we present the optimization and voting models. The problems are defined in a network with nodes representing voters having similar "values". There is a set of candidates, and the distance from a node to each candidate is a proxy for the utility obtained from that candidate in a particular dimension. There is a