European Journal of Operational Research 253 (2016) 204–215

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

On information-based warranty policy for repairable products from heterogeneous population

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ARTICLE INFO

Article history:
Received 21 October 2015
Accepted 11 February 2016
Available online 18 February 2016

Keywords:
Warranty policy
Operational history
Repairable item
Mixed population
Stochastically ordered subpopulations

ABSTRACT

Preventive maintenance over the warranty period has a crucial effect on the warranty servicing cost. Numerous papers on the optimal maintenance strategies for warranted items have focused on the case of items from homogeneous populations. However, most of real life populations are heterogeneous. In this paper, we assume that an item is randomly selected from a mixed population composed of two stochastically ordered subpopulations and that the subpopulation, from which the item is chosen, is unknown. As the operational history of an item contains the information on the chosen subpopulation, we utilize this information to develop and justify a new information-based warranty policy. For illustration of the proposed model, we provide and discuss relevant numerical examples.

1. Introduction

The warranty is an important contract between a manufacturer and a buyer and requires the manufacturer to have the responsibility to repair or replace all failures occurring during a prespecified period. Accordingly, a proper warranty strategy is fundamental for manufacturer to reduce the warranty servicing costs. Blischke and Murthy (1992) presented the concept and theoretical models for basic warranty. Of particular interest are the early papers of Biedenweg (1981), Nguyen and Murthy (1986) and Murthy and Nguyen (1988), where the optimal strategy is proposed by using the technique of dividing the warranty period into distinct intervals according to replacement and repair actions. For instance, in Biedenweg (1981), the failed items are replaced (by the manufacturer) by the new ones from the time of purchase until a prescribed time, and any further failures occurring during the remainder of the warranty period are rectified by a repair action. More comprehensive and extensive discussion on the optimal replace-repair strategy under warranty can be found, e.g., in Jack and Van der Duyn Schouten (2000), Zuo, Liu, and Murthy (2000) and Jack and Murthy (2001) to name a few.

One possible way of reducing the warranty servicing costs is to incorporate effective preventive maintenance (PM) actions into the warranty policy. For deteriorating and repairable items, PM actions are performed at a planned time(s) to make the item 'younger' while it is still in the operating state. From the manufacturer's perspective, although the PM actions impose additional costs, the item's improvement due to the PM actions could reduce the warranty servicing costs over the warranty period. The warranty models with PM have been intensively studied in the literature (see, e.g., Sahin & Polatoglu, 1996; Monga & Zuo,1998; Yeh & Lo, 2001; Jack & Murthy, 2002; Kim, Djamaludin, & Murthy, 2004; Chien, 2008; Chang & Chien, 2012; Yeh, Kurniati, & Chang, 2015; Chien & Zhang, 2015; Liu, Wu, & Xie, 2015; Huang et al., 2015; Wang, Liu, & Liu, 2015).

Throughout this paper, we will deal with the warranty policy with PM from the manufacturer's perspective. Our goal is to define optimal maintenance strategies under warranty which minimize the warranty servicing costs. To the best of our knowledge, most of research on the warranty with PM modeling was performed under the assumption that a population of items is homogeneous with respect to reliability characteristics, i.e., that their lifetimes can be described by the same probability distribution. However, the real life populations are usually heterogeneous. Therefore, it is quite a challenge to incorporate heterogeneity into the framework of warranty modeling. Note that the concept of heterogeneity has been a central issue in many disciplines such as biology, ecology, demography, survival analysis and reliability modeling (see, e.g., Dushoff & Levin, 1995; Kendall, Gordon, Fujiwara, & Nogeire, 2011; Keyfitz & Casewell, 2005; Vaupel & Yashin, 1985; Badía, Berrade, & Campos, 2003; Cha & Finkelstein, 2011; Finkelstein & Cha, 2013).

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http://dx.doi.org/10.1016/j.ejor.2016.02.020
0377-2217/

In this paper, we propose a new warranty policy for items from a heterogeneous population. The item is randomly selected from a mixed population which consists of two stochastically ordered subpopulations and it is assumed that the subpopulation from which the item is selected is unknown. Here, by the 'two stochastically ordered subpopulations', we mean that the lifetime of items from one subpopulation is larger (or smaller) than that of the other subpopulation in a suitable stochastic sense. Operation history of an operating repairable item (e.g., its sequential failure times) obviously depends on the subpopulation from which it has been chosen. This operational history of the repairable items plays an important role in providing important information about reliability characteristics of items, which in turn helps us to establish a more effective maintenance strategy under warranty. While there exists literature which utilizes the historical data to estimate the degradation of the product and make warranty-related decisions (see, e.g., Fang & Huang, 2010; Guida, Pulcini, & Vainello, 2009; Majeske, 2007; Silver & Fiechter, 1995), in this paper, we use a different type of information. We apply different maintenance actions during warranty to the items depending on the classification of an item as belonging to a weak or strong. This classification is performed using the operational history data. We suggest and discuss the corresponding information-based warranty policy and show that it is more cost-effective than the conventional one that does not use operational information.

This paper is organized as follows. In Section 2, the population structure is described and the information-based warranty policy is suggested. Under the proposed warranty policy, the general expression for the warranty servicing cost function is also derived and the two-stage optimization procedure is briefly discussed. In Section 3, a generalization of the proposed warranty policy is considered. In Section 4, several numerical examples are provided for illustration. Furthermore, the extension of the proposed information-based warranty policy to the case of multiple PMs is discussed in Section 5. Finally, in Section 6, the concluding remarks are given.

2. Information-based warranty policy

2.1. Population structure and conventional warranty policy

In this section, we will provide a 'general formulation' for the information-based warranty policy. Suppose that a population of items is a mixture of two stochastically ordered subpopulations. The subpopulation of items with (stochastically) shorter lifetimes will be, just for convenience, called the 'weak subpopulation', whereas that of items with relatively longer lifetimes will be called the 'strong subpopulation'. Denote the lifetime of an item from the weak subpopulation by T_W and its absolutely continuous Cdf, pdf and the failure rate function by $F_1(t)$, $f_1(t)$ and $\lambda_1(t)$, respectively. Similarly, the lifetime, the Cdf, pdf and the failure rate function of an item from the strong subpopulation are denoted by T_S, $F_2(t)$, $f_2(t)$ and $\lambda_2(t)$, accordingly. Mathematical definitions of weak and strong subpopulations will be given after introducing the notation. The initial $(t = 0)$composition of our mixed population is as follows: the proportion of the weak items is π, whereas the proportion of the strong items is $1-\pi$, which means that the corresponding frailty variable Z (Finkelstein & Cha, 2013) in this case has a discrete probability distribution:

$$\pi(z) = \begin{cases} \pi, \ z=1, \\ 1-\pi, \ z=2, \end{cases} \tag{1}$$

where $z = i$ indicates the 'weak' and 'strong' subpopulations, $i = 1, 2$, respectively. For convenience, in the following, we will let $\pi_1 \equiv \pi$ and $\pi_2 \equiv 1-\pi$. The mixture (population) survival function denoted by $\bar{F}_m(t)$ is

$$\bar{F}_m(t) = \pi_1 \bar{F}_1(t) + \pi_2 \bar{F}_2(t), \tag{2}$$

where $\bar{F}_i(t) = 1 - F_i(t), i = 1, 2$. Then the mixture (the observed or the population) failure rate is

$$\lambda_m(t) = \frac{\pi_1 f_1(t) + \pi_2 f_2(t)}{\pi_1 \bar{F}_1(t) + \pi_2 \bar{F}_2(t)} = \pi_1(t)\lambda_1(t) + \pi_2(t)\lambda_2(t),$$

where the corresponding time-dependent probabilities are

$$\pi_1(t) = \frac{\pi_1 \bar{F}_1(t)}{\pi_1 \bar{F}_1(t) + \pi_2 \bar{F}_2(t)}, \pi_2(t) = \frac{\pi_2 \bar{F}_2(t)}{\pi_1 \bar{F}_1(t) + \pi_2 \bar{F}_2(t)}.$$

Assume further that the subpopulations are ordered (and thus, the weak and strong subpopulations are now mathematically defined) in the sense of the failure rate ordering (see, e.g., Shaked & Shanthikumar, 2007).

$$\lambda_1(t) > \lambda_2(t), \ \ t > 0. \tag{3}$$

Furthermore, for deteriorating items, it is reasonable to assume that the failure rates corresponding to each subpopulation are strictly increasing. Then, in principle, the cost-effectiveness of the PM during the warranty period could be justified.

It should be noted that, in practice, most often, populations of manufactured items are composed of two ordered subpopulations: the subpopulation with normal lifetimes (Main (Strong) Distribution) and the subpopulations with relatively shorter lifetimes (Weak Distribution). Items belonging to the 'weak distribution' can be produced along with the items of the 'main distribution' due to, for example, defective resources and components, human errors, unstable production environment caused by uncontrolled significant quality factors, etc. (See, Jensen & Petersen, 1982; Kececioglu & Sun, 2003).

Our approach generalizes the conventional warranty policy developed for items from homogeneous populations (see, e.g., Biedenweg,1981) that does not consider the operational history to the case of heterogeneous populations taking into account the corresponding operational history of items. The latter is a major challenge of this paper.

Our setting is as follows: A new item is randomly chosen from the mixed population described above and it is assumed that the subpopulation from which the item is selected is unknown. We also assume that under the non-renewing warranty with the warranty period W, the item is *minimally repaired* at all failures and preventively maintained at a fixed time $T < W$ without any cost to the buyers in $[0, W]$. Thus we consider here a free repair/replacement, one-dimensional warranty policy. Under these assumptions, we will derive the corresponding expected warranty servicing cost.

Let c_m be the cost of minimal repair and c_{PM} be the cost of the PM. To motivate the information-based warranty policy to be discussed in Section 2.2 and also for further comparison, we will now briefly discuss two conventional warranty policies which do not take into account operational history of items:

Option 1: no preventive maintenance during the warranty period $[0, W)$.

Option 2: preventive maintenance at time T during $[0, W)$.

Let us define first the expected warranty servicing costs during the warranty period $[0, W)$. These quantities are obviously given for Option 1 and Option 2 as

$$C_0 \equiv c_m E[N(W)] \tag{4}$$

and

$$C_1(T) \equiv c_m E[N(T)] + c_{PM} + c_m E[N_{PM}(W-T)], \tag{5}$$