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Complex multi-state systems modelled through marked Markovian arrival processes

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ABSTRACT

Complex multi-state warm standby systems subject to different types of failures and preventive maintenance are modelled by considering discrete marked Markovian arrival processes. The system is composed of K units, one online and the rest in warm standby and by an indefinite number of repairpersons, R. The online unit passes through several performance states, which are partitioned into two types: minor and major. This unit can fail due to wear or to external shock. In both cases of failures, the failure can be repairable or non-repairable. Warm standby units can only undergo repairable failures due to wear. Derived systems are modelled from the basic one according to the type of the failure; repairable or non-repairable, and preventive maintenance. When a unit undergoes a repairable failure, it goes to the repair facility for corrective repair, and if it is non-repairable, it is replaced by a new, identical one. Preventive maintenance is carried out in response to random inspections. When an inspection takes place, the online unit is observed and if the performance state is major, the unit is sent to the repair facility for preventive maintenance. Preventive maintenance and corrective repair times follow different distributions according to the type of failure. The systems are modelled in transient regime, relevant performance measures are obtained, and rewards and costs are calculated. All results are expressed in algorithmic form and implemented computationally with Matlab. A numerical example shows the versatility of the model presented.

1. Introduction

Redundant systems and preventive maintenance, which are two ways to improve system reliability and availability, are of considerable research interest. Serious damage, financial losses and, possibly, total system failure can be provoked by poor reliability. Two approaches can be adopted to improve the reliability of a complex system: standby systems and preventive maintenance. Various classes of redundant systems have been proposed, depending on the problem to be addressed. Levitin, Xing, and Dai (2014) developed an optimisation problem, in which a fixed set of elements was distributed between cold and warm standby groups; an appropriate element initiation sequence was then selected to minimise the expected mission operation cost of the system while providing the desired level of system reliability. In this respect, too, Vanderperre and Makhanov (2014) analysed a repairable duplex system characterised by cold standby and by pre-emptive priority rules. In this paper, general probability distributions for failure and repair were allowed. There exists an extensive body of literature related to warm standby systems, from significant initial research such as that by Gnedenko (1965), who analysed a warm standby system with a general number of components, to recent papers such as Wells (2014) where known analytic results are extended to a case with repairable and non-repairable failures.

Preventive maintenance is intended to improve system reliability and to increase profits. Nakagawa (2005) studied standard and advanced problems of maintenance policies for system reliability. Zhong and Jin (2014) included preventive maintenance in a cold standby two-component system, using semi-Markovian processes. In order to keep components running properly, the working component receives periodic preventive maintenance. An optimal replacement policy was developed by Zhang and Wang (2011) to cope with a deteriorating system with multiple types of failures. Under this approach, the application of an optimal replacement policy ensures that the long-run expected reward per unit of time is maximised. Preventive maintenance has also been described for use in complex systems, with either a multi-state unit or with a general set of cold standby multi-state units (Ruiz-Castro, 2013, 2014).

Nowadays, multi-state systems are of particular importance in ensuring reliability. Most texts on reliability theory analyse systems in which the units perform in terms of traditional binary

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models: up state (performing) and down state (failure). Many real-life systems, termed multi-state systems, are composed of multiple components with different performance levels and incorporating several failure modes. In this respect, Natvig and Morch (2003) analysed the Norwegian offshore gas pipeline network in the North Sea, transporting gas to Emden in Germany, and Lisnianski, Frenkel, and Ding (2010) have studied multi-state systems, presenting a variety of significant cases of interest to engineers and industrial managers. Lisnianski and Frenkel (2012) included Markov processes in the analysis of multi-state systems, highlighting the benefits of their application.

When complex systems are modelled, intractable expressions are often encountered. Several methodologies have been proposed to analyse the behaviour of a multi-state system, and one such method is that of Markov process theory. Markov processes enable us to model the behaviour of a complex multi-state system and to obtain measures in an algorithmic and computational form. One class of distributions that makes it possible to model complex systems with well structured results, thanks to its matrix-algebraic form, is the phase-type distribution (PH), which was introduced and analysed in detail by Neuts (1975, 1981), who pointed out its useful algorithmic properties. Phase type distributions and Markov processes have been applied in fields such as queuing theory, survival and reliability, where real-life problems have been modelled in an algorithmic form (Ruiz-Castro & Fernández-Villodre, 2012).

Many stochastic systems have inputs to the system over time that can be counted to control events, e.g. electrical systems at which electric shock waves arrive at random intervals. Multi-state systems that evolve over time may be subject to different types of failures, whether repairable or non-repairable, and benefit from measures such as preventive maintenance to enhance performance and economic results. The analysis of these systems requires a mathematical tool that can describe the input analytically and give rise to a numerically tractable model. The Markovian arrival process (MAP) class that was introduced by Neuts (1979) counts the number of events in an underlying Markov chain. Two special cases of this process are Batch MAP and Marked MAP. In the first case, arrivals in batch are allowed, and in the second, several types of arrivals are counted. In all cases, the arrival rates of events can be customised for different situations, which highlight the inherent versatility of this class of processes. In a recent study, He (2014) presented the main results associated with MAPs.

Reliability systems are usually studied in the continuous case. However, not all systems can be continuously monitored, and some must be observed at certain times, for reasons such as the internal structure of the system, the need for periodic inspections, etc. Reliability systems that evolve in discrete time have been proposed to analyse the behaviour of devices in fields such as civil and aeronautical engineering. Ruiz-Castro and Quan-Lin (2011) considered a Markovian structure to model a k-out-of-n: G system with multi-state components by means of well-structured blocks. Recently, Ruiz-Castro (2014) included preventive maintenance in a discrete system to analyse its effectiveness with respect to performance measures and related costs in a complex device.

The aim of the present paper is to model certain warm standby complex systems that evolve in discrete time, are subject to different types of failure (repairable and non-repairable) and are protected by means of preventive maintenance with an indeterminate number of repairpersons. External shocks are included and from a basic system various complex ones are derived. This evolution is analysed using a Markov model and the main measures are determined in an algorithmic and computational form. Events occur at different times and are modelled by a marked, batch-arrival MAP. Costs are introduced and a numerical example shows the versatility of the modelling, comparing two similar complex systems with and without preventive maintenance. Various measures are applied to determine whether preventive maintenance is profitable from performance and financial standpoints. The results presented in this paper were obtained in an algorithmic and computational form, through the use of this methodology.

Section 2 presents the basic system, the assumptions and the state space, and Section 3 describes Marked MAPs in detail and obtains the matrix blocks and the transition probability matrix that describes the evolution of the system. Section 4 then addresses the modelling of the four systems derived. Section 5 is focused on the performance measures; thus, availability, reliability, conditional probability of failure, mean times and mean number of events are determined in transient regime. Section 6 introduces the concept of rewards, together with measures such as mean costs and profit up to a certain time. Finally, the versatility of the modelling is shown in Section 7, with a comparison of two similar systems, with and without preventive maintenance.

2. The basic system (system I)

We assume a K-system with the online unit and the rest in warm standby that evolves in discrete time. The online unit is subject to repairable or non-repairable internal failures due to wear out. Also, the online unit is subject to random external events which can produce external shocks by producing failure. This one can be repairable and non-repairable depending on time up to failure. Any warm standby unit can undergo only repairable failures due to wear. When one failure occurs, the unit goes to the repair facility for corrective repair. The repair facility is composed of an indefinite number of repairpersons R where $R \leq K$. The online unit is a multi-state one where it passes through several performance stages which are partitioned in minor (the first n_1 states) and major (the rest). Inspections occur randomly and in response to these ones preventive maintenance can be carried out. The online unit goes to preventive maintenance only when one major state is observed under inspection. Corrective repair times are different according to the type of failure, from either the online place or standby. The order of the type of failure in queue keeps in memory.

2.1. The assumptions

The system described above is subject to the following assumptions.

Assumption 1. The internal operational time of the online unit is *PH*-distributed with representation $(\boldsymbol{\alpha}, \mathbf{T})$. The number of operational states is equal to n, and these are partitioned in minor (the first n_1 states) and major states (states $n_1 + 1, \ldots, n$).

Assumption 2. Internal failures can be repairable and non-repairable. When an internal repairable failure occurs, a transition occurs to a subset of states, and the same happens for the non-repairable failures with another subset of states. The internal time up to failure can be written by blocks as $(\mathbf{T}|\mathbf{T}_r^0|\mathbf{T}_{nr}^0)$ where the blocks $\mathbf{T}_r^0$ and $\mathbf{T}_{nr}^0$ are column vectors including the absorbing probabilities from the transient states for an internal repairable and non-repairable failure, respectively. The absorbing probabilities for an internal failure from the transient states are given by the column vector $\mathbf{T}^0 = \mathbf{T}_r^0 + \mathbf{T}_{nr}^0$.

Assumption 3. Events that produce failures of the online unit due to external shocks occur according to a phase type renewal process. If the online place is occupied, this event produces the failure of the unit. The time between two consecutive events is *PH* distributed with representation $(\boldsymbol{\gamma}, \mathbf{L})$. The order of the matrix $\mathbf{L}$ is equal to t.