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Discrete Optimization

Advanced Greedy Randomized Adaptive Search Procedure for the Obnoxious p-Median problem☆

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ABSTRACT

The Obnoxious p-Median problem consists in selecting a subset of p facilities from a given set of possible locations, in such a way that the sum of the distances between each customer and its nearest facility is maximized. The problem is $\mathcal{NP}$-hard and can be formulated as an integer linear program. It was introduced in the 1990s, and a branch and cut method coupled with a tabu search has been recently proposed. In this paper, we propose a heuristic method – based on the Greedy Randomized Adaptive Search Procedure, GRASP, methodology – for finding approximate solutions to this optimization problem. In particular, we consider an advanced GRASP design in which a filtering mechanism avoids applying the local search method to low quality constructed solutions. Empirical results indicate that the proposed implementation compares favorably to previous methods. This fact is confirmed with non-parametric statistical tests.

1. Introduction

Facility location has been of practical and theoretical interest for more than the half of a century (Cohen, 1973; Klose & Drexl, 2005). First linear programming (LP) formulations origin in the late fifties and were soon followed by solution techniques, which later on became well-known as Branch and Bound (B&B) or Mixed Integer Programing (MIP). The classical so-called warehouse location problem (WLP), also well-known as (simple) facility or plant location problem, is an uncapacitated optimization problem, which can be modeled as an LP (Balinski, 1965) or network problem (Drezner & Hamacher, 2004; Melkote & Daskin, 2001). It is a site-selecting location-allocation model with a min–sum objective, where from a number of potential facility or warehouse sites the set of costumers has to be serviced, while minimizing the total fixed site-costs (location) plus the total variable customer assignment costs (allocation). The WLP is $\mathcal{NP}$-hard (Garey & Johnson, 1979; Papadimitriou & Yannakakis, 1991) and, as such, has been in the focus of researchers interested in developing specialized LP-approaches (Körkel, 1999) or approximative constructive and local search or improvement algorithms, including standard add- or drop-heuristics, and Lagrangian approaches (Beasley, 1993; Kuehn & Hamburger, 1963), which then were followed by more advanced metaheuristics like genetic algorithms or tabu search and its derivatives (Greistorfer & Rego, 2006; Kratica, Tošić, Filipović, & Ljubic, 2001; Michel & Hentenryck, 2004). Comparing the WLP with the p-median problem, pMP (Hakimi, 1964; 1965), one identifies two differences: (1) there are no fixed site-costs involved and (2) the number of finally opened sites, p, is no longer a decision variable, but becomes included in the model. The pMP can be solved in polynomial time for fixed values of p, but is strongly $\mathcal{NP}$-hard for variable values of p (Current, Daskin, and Shilling, 2004, chap. 3; Garey & Johnson, 1979; Megiddo & Supowit, 1984). Consequently, any pMP is a special case of the general class of WLPs. Such as for the WLP, a variety of solution procedures, exact approaches, primal-dual approaches and metaheuristics, have been introduced for the pMP (Mladenović, Brimberg, Hansen, & Moreno-Pérez, 2007; Reese, 2006; Tansel, Francis, & Lowe, 1983). A most recent paper, Batta, Lejeuneb, and Prasad (2014), is recommended for a classical as well as modern view (dispersion, population, and equity criteria) on locational developments, offering an additional special focus on pMP.

This work relates to location type problems like the WLP and pMP, additionally taking account of the so-called obnoxious or semi-obnoxious effects. Such effects often occur when interesting services of some provider are based on unwanted, but inevitably necessary locations, which do not add additional value to the

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Table 1
Matrix of distances. Last row shows the sum of all the client distances for each facility.

	j_1	j_2	j_3	j_4	j_5	j_6
i_1	3	2	10	13	4	8
i_2	14	14	11	3	15	12
i_3	5	4	6	12	14	6
i_4	3	1	1	3	10	9
i_5	13	9	2	10	9	4
i_6	9	5	11	14	4	11
i_7	4	3	4	13	4	2
i_8	12	2	9	3	15	13
i_9	9	11	1	2	7	4
$\sum$	72	51	55	73	82	69

Table 2
Example of solutions with $p = 3$.

(a) $S = \{j_2, j_5, j_6\}$				(b) $S' = \{j_2, j_3, j_4\}$			
	j_2	j_5	j_6		j_2	j_3	j_4
i_1	**2**	4	8	i_1	**2**	10	13
i_2	14	15	**12**	i_2	14	11	**3**
i_3	**4**	14	6	i_3	**4**	6	12
i_4	**1**	10	9	i_4	**1**	1	3
i_5	9	9	**4**	i_5	9	**2**	10
i_6	5	**4**	11	i_6	**5**	11	14
i_7	3	4	**2**	i_7	**3**	4	13
i_8	**2**	15	13	i_8	**2**	9	3
i_9	11	7	**4**	i_9	11	**1**	2
		$f(S) = 35$				$f(S) = 23$	

product. Quite contrary, pure obnoxiousness clearly devalues the production process. Obnoxious problems were firstly coined in Church and Garfinkel (1978) who located a facility in a network using an exact method and also introduced the term of the so-called bottleneck points for a network.

Erkut and Neuman (1989) used the terms *disservice* and *service* of the people in the vicinity of an (semi-)obnoxious facility that occurs during the processing of a product. In general, such services include a type of hazardous material, waste disposal, water treatment, nuclear power or chemical plants as well as big public facilities like airports. Regularly, these problems arise in the context of urban settings when the network may consist of noisy or polluting roads, transportation corridors, or rail lines (Segal, 2003).

The present work deals with the Obnoxious p-Median (OpM) problem. It can be formally defined as follows. Let I be a set of clients, J a set of facilities, and d_{ij} the distance between the client $i \in I$ and the facility $j \in J$. The OpM problem consists in finding a set S with p facilities (with $S \subseteq J$ and $p < |J|$), such that the sum of the minimum distance between each client and the set of facilities is maximized. In mathematical terms:

$$\max \sum_{i \in I} \min\{d_{ij} : j \in S\}$$

subject to

$$S \subseteq J \ , \ |S| = p$$

Facilities in S are called open facilities, while facilities in $J \backslash S$ are known as closed or unopened facilities.

Table 1 shows an example of pair-client distances, where there are 9 clients $I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9\}$ and 6 potential facilities $J = \{j_1, j_2, j_3, j_4, j_5, j_6\}$. Suppose that $p = 3$. Then, a solution of the OpM consists of selecting 3 facilities out of 6 in J. Table 2a shows a solution with $S = \{j_2, j_5, j_6\}$. The minimum distance between each client and the corresponding facility is highlighted with bold font. For example, the distances from client i_1 to each facility are: $d(i_1, j_2) = 2$, $d(i_1, j_5) = 4$, and $d(i_1, j_8) = 8$. Then, j_2 is the closest facility to client i_1. The value of the objective function of this solution, denoted as $f(S)$, is the sum of the minimum distances from each client to the set of facilities: $f(S) = 2 + 12 + 4 + 1 + 4 + 4 + 2 + 2 + 4 = 35$.

Table 2 b shows a different solution $S' = \{j_2, j_3, j_4\}$, where the value of the objective function is $f(S') = 23$. Considering that the OpM is a maximization problem, solution S is better than solution S'. In other words, the minimum distances among the clients and facilities in S has a sum that is larger than the one in S'.

In this paper we explore the adaptation of the Greedy Randomized Adaptive Search Procedure methodology, GRASP, introduced in Feo and Resende (1989), to solve the OpM problem. Each GRASP iteration consists of constructing a trial solution and then applying an improvement procedure to find a local optimum. We explore different designs for both phases, construction and improvement. In particular, in Section 3 we propose two different constructive methods and in Section 4, we describe two local search algorithms. Additionally, we propose an efficient strategy to update the objective function value that considerably reduces the running time of GRASP. In Section 5, we present a filtering strategy intended to discard low-quality solutions and to selectively apply the local search only to promising solutions. This mechanism reduces the computing time without deteriorating the quality of the final solution. Section 6 reports on an extensive computational experience to validate the proposed algorithm by comparing its performance with those in the current state of the art. Finally, Section 7 summarizes the main conclusions of our research.

2. Literature review

There exists an extensive literature treating obnoxious and semi-obnoxious situations, modeled on the plane or in a graph, using max–min, max–max and often bi-criteria objectives (Batta et al., 2014; Cappanera, 1999; Conceição Fonseca & Captivo, 2007; Erkut & Neuman, 1989; Farahani, SteadieSeifi, & Asgari, 2010; Segal, 2003). This goes along with the vast research on (obnoxious) p-median, p-center and similar location problems. Some of these sources are highlighted subsequently.

In Drezner and Wesolowsky (1980) a planar, Euclidean model of a 1-facility-problem is presented in which the shortest weighted distance to a point is maximized. Simultaneously, a side constraint must hold, i.e., the facility must be within a pre-specified distance from each point. The problem is solved using a graphical circles-based approach. A continuation of this work can be found in Drezner and Wesolowsky (1983), where a constrained obnoxious problem is considered for which a weighted rectangular-metric objective has to be maximized. The authors propose two algorithms, a so-called boundary and segment search and an LP-based approach. A semi-obnoxious scenario is solved in Melachrinoudis (1985). It uses a max–min model and seeks for a point on a convex two-dimensional bounded region, which maximizes the minimum weighted distance from that point to a given set of existing points in the region to be considered. As before, a circles-based, straightforward geometrical approach as well as an exact algorithm, using Kuhn–Tucker boundary points, is introduced. Complexity issues regarding the placement of several facilities in an obnoxious setting, a p max–min problem, were considered by Tamir (1991). It is shown that even the finding of an approximate solution is $\mathcal{NP}$-hard. Again, Drezner and Wesolowsky (1996) deal with an Euclidean network and an obnoxious urban situation in which a location has to be determined inside the convex hull of a number of nodes in order to maximize the minimum weighted distance between this point and the nodes and arcs of the network. Among others, the authors offer an ϵ-approximation scheme, which was later improved in Segal (2003). Welch and Salhi (1997) present three heuristics to solve the max–min formulation for siting p