European Journal of Operational Research 252 (2016) 466–476

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

A hybrid scenario cluster decomposition algorithm for supply chain tactical planning under uncertainty

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ARTICLE INFO

Article history:
Received 13 August 2015
Accepted 23 January 2016
Available online 30 January 2016

Keywords:
Multi-stage stochastic programming
Scenario decomposition
Lagrangian Relaxation
Sub-gradient method
Variable Fixing Heuristic

ABSTRACT

We propose a Hybrid Scenario Cluster Decomposition (HSCD) heuristic for solving a large-scale multi-stage stochastic mixed-integer programming (MS-MIP) model corresponding to a supply chain tactical planning problem. The HSCD algorithm decomposes the original scenario tree into smaller sub-trees that share a certain number of predecessor nodes. Then, the MS-MIP model is decomposed into smaller scenario-cluster multi-stage stochastic sub-models coordinated by Lagrangian terms in their objective functions, in order to compensate the lack of non-anticipativity corresponding to common ancestor nodes of sub-trees. The sub-gradient algorithm is then implemented in order to guide the scenario-cluster sub-models into an implementable solution. Moreover, a Variable Fixing Heuristic is embedded into the sub-gradient algorithm in order to accelerate its convergence rate. Along with the possibility of parallelization, the HSCD algorithm provides the possibility of embedding various heuristics for solving scenario-cluster sub-models. The algorithm is specialized to lumber supply chain tactical planning under demand and supply uncertainty. An ad-hoc heuristic, based on Lagrangian Relaxation, is proposed to solve each scenario-cluster sub-model. Our experimental results on a set of realistic-scale test cases reveal the efficiency of HSCD in terms of solution quality and computation time.

1. Introduction

Large-scale Multi-stage Stochastic Mixed Integer Programming (MS-MIP) models usually arise in multi-period planning problems under uncertain parameters with dynamic and non-stationary behavior over the planning horizon. Supply chain planning (e.g., Alonso-Ayuso, Escudero, Garín, Ortuño, & Pérez, 2003; Alonso-Ayuso, Escudero, & Ortuño, 2003), and production planning (Kazemi Zanjani, Ait-Kadi, & Nourelfath, 2013; Kazemi Zanjani, Nourelfath, & Ait-Kadi, 2010; Kazemi Zanjani, Nourelfath, & Ait-Kadi, 2013) under uncertainty are few examples among others.

Such models are among the most intractable ones due to the fact that the number of complicating binary and/or integer variables in the deterministic MIP model increases exponentially once the uncertainty is modeled as a scenario tree in a multi-stage setting. The latter is a viable way of capturing the evolution of all information trajectories over time. A variety of algorithms for solving multi-stage stochastic MIP models have been proposed (e.g., see van der Vlerk, 1996). Branch-and-Price (Lulli & Sen, 2004) and Branch-and-Fix Coordination methods (Alonso-Ayuso, Escudero, Garín et al., 2003; Alonso-Ayuso, Escudero, & Ortuño, 2003; Escudero, Garín, Merino, & Pérez, 2010; 2012)) are two of prevalent methods in the literature for solving MS-MIP models. Nonetheless, such algorithms are designated for special structured models such as lot-sizing and batch-sizing problems or pure 0–1 integer programming models. This makes them less suitable for general large-scale MS-MIP models with no particular structure similar to the supply chain tactical planning model investigated in this article.

Scenario decomposition strategies (e.g., see Carøe & Schultz, 1999; Rockafellar & Wets, 1991) are among the most efficient approaches to solve large-scale multi-stage stochastic programs. Progressive Hedging Algorithm (PHA) (Rockafellar & Wets, 1991) is one of the scenario decomposition techniques that has been successfully applied as a heuristic to solve multi-stage stochastic MIP models. The main idea behind this algorithm is to decompose the original multi-stage stochastic program into deterministic scenario sub-models. Such subproblems are then coordinated by Lagrangian penalty terms in their

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http://dx.doi.org/10.1016/j.ejor.2016.01.048

Nomenclature

Sets

BL	set of harvesting blocks
M	set of manufacturing mills
RM	set of raw materials
SC	set of available raw material scenarios
t_n	set of time periods corresponding to node n in the scenario tree
Tree	the demand scenario tree

Parameters

$a(n)$	set of immediate predecessor of each node n in scenario tree
b_t^H	the total harvesting capacity in period t
b_t^T	the total transportation capacity in period t
c_{blt}^H	unit cost to harvest block bl during period t
$c_{rm,bl,m,t}^T$	unit cost to transport raw material rm from block bl to mill m during period t
$c_{rm,bl,t}^S$	unit cost to store raw material rm in block bl during period t
$f_{rm,\ bl,\ t}$	stumpage fee for raw material rm in block bl during period t
$h_{rm,\ m}$	inventory holding cost of raw material rm at mill m
$KI_{m,\ t}$	inventory capacity at mill m during period t
KS_t^{bl}	supply capacity of block bl in period t
l_{bl}	maximum number of periods over which harvesting can occur in block bl
L_{rm}^{bl}	lead time of procuring raw material rm from block bl
$m_{rm,t}^{bl}$	unit purchase cost of raw material rm from block m in period t
n_t	maximum number of blocks in which harvesting can occur during period t
$p(n)$	probability of node n in scenario tree
p_{sc}	probability of supply scenario sc
$qmin^{bl}$	minimum contract purchase quantity from block bl
$ss_{rm,\ m}$	safety stock of raw material rm at mill m
$v_{rm,bl,sc}$	volume of available raw material rm in block bl for supply scenario sc
$d_{rm,\ m,\ t}(n)$	forecasted demand of raw material rm for mill m in period t at node n of the scenario tree

Decision variables

$H_{bl,\ t}(n)$	binary variable that takes 1 if harvesting occurs in block bl during time period t at node n of the scenario tree and 0 otherwise
$I_{rm,\ bl,\ t,\ sc}(n)$	inventory of raw material rm in block bl at the end of period t for supply scenario sc at node n of the scenario tree
$I_{rm,\ m,\ t,\ sc}(n)$	inventory of raw material rm at mill m at the end of period t for supply scenario sc at node n of the scenario tree
$X_{rm,m,t}^{bl}(n)$	purchasing quantity of raw material rm from block bl in period t at node n of the scenario tree
$y_{bl,\ t}(n)$	proportion of harvested block bl in period t at node n of the scenario tree

objective function in order to obtain an implementable solution. Løkketangen and Woodruff (1996) proposed a heuristic algorithm based on PHA and Tabu Search. Haugen, Løkketangen, and Woodruff (2001) cast the PHA in a meta-heuristic algorithm where the generated sub-problems for each scenario are solved heuristically. Despite several advantages of Scenario Decomposition (SD) algorithms in solving stochastic programs, such approaches suffer from critical issues non-convergence or unacceptably long run-times in the context of large-scale MS-MIP models. Recently, Watson and Woodruff (2011) proposed algorithmic innovations to address several critical issues of PHA in the context of large-scale two-stage discrete optimization problems. In an attempt to speed up scenario decomposition algorithms in the context of large-scale MS-MIP models, the idea of scenario partitioning (clustering) in scenario trees has been proposed by several authors. The idea is to decompose the initial scenario tree into smaller sub-trees that share a certain number of ancestor nodes. The multi-stage stochastic model is then decomposed into scenario cluster sub-models which are coordinated by Lagrangian penalty terms in their objective function in order to compensate the lack of non-anticipativity. Escudero et al. (2010) embedded the idea of scenario partitioning with the Branch-and-Fix Coordination method to solve large-scale 0–1 multi-stage stochastic models. Escudero, Garín, and Unzueta (2016) proposed a cluster Lagrangian decomposition algorithm for solving MS-MIP model while implementing four approaches for updating Lagrangian multipliers. Carpentier, Gendreau, and Bastin (2013) proposed a heuristic for scenario partitioning of large scenario trees within the PHA. Escudero et al. (2016), Escudero, Garín, Pérez, and Unzueta (2013) demonstrated that adopting the sub-gradient algorithm to coordinate scenario cluster sub-models would result considerably higher convergence rates comparing to the PHA. It is noteworthy that in the PHA, an implementable solution in each node of the scenario tree is considered as the average of solutions of the set of scenario cluster sub-models that are indistinguishable at that node. In contrary, the sub-gradient method directly imposes the implementability condition in each node through a pair-wise comparison between the solutions of the set of indistinguishable scenario clusters at that node.

In this study, *we extend the idea of scenario clustering of Escudero et al. (2016) based on an accelerated sub-gradient method*. More precisely, we propose a scenario cluster decomposition (SCD) based on the sub-gradient method. Along with this contribution, there are also three other main contributions in the present paper. First, with the goal of reducing the number of iterations, *we embed a variable-fixing heuristic within the sub-gradient algorithm*. This algorithm fixes the value of binary variables in the common nodes of scenario cluster sub-models obtained at each iteration of SCD algorithm to zero or one in the next iteration according to a consensus rule among the solution of indistinguishable scenario cluster sub-models. Second, *the accelerated SCD algorithm is specialized to tactical supply and procurement planning in the lumber supply chain under demand and supply uncertainty*. To the best of our knowledge, due to high computational complexity, this problem has never been addressed in the literature. In this problem, scenario-cluster sub-models are MS-MIP models that are hard to solve. Hence, our third contribution is focused on *proposing an ad-hoc heuristic to solve such sub-models*. This algorithm is a Lagrangian Relaxation-based heuristic enhanced through updating the sub-gradient step-size.

Hence, the proposed algorithm in this study is a Hybrid Scenario Cluster Decomposition (HSCD) heuristic applicable to large-scale MS-MIP models with a particular application in supply chain tactical planning. Along with the possibility of parallelization, the main advantage of the HSCD heuristic is accelerating the sub-gradient algorithm applied to coordinate scenario cluster sub-models into an implementable solution. Furthermore, it provides the possibility of embedding proper heuristics for solving scenario cluster sub-models depending on their special structure. Hence, significant improvement in terms of the convergence rate of the