European Journal of Operational Research 252 (2016) 79–89

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

# Optimal restricted due date assignment in scheduling

Dvir Shabtay*

*Department of Industrial Engineering and Management, Ben-Gurion University of the Negev, Beer Sheva, Israel*

ARTICLE INFO

*Article history:*
Received 3 December 2014
Accepted 22 December 2015
Available online 31 December 2015

*Keywords:*
Scheduling
Due date assignment
Approximation algorithms
Earliness
Tardiness

ABSTRACT

In classical scheduling problems it is common to assume that the due dates are predefined parameters for the scheduler. In integrated systems, however, due date assignment and scheduling decisions have to be carefully coordinated to make sure that the company can meet the assigned due dates. Thus, a huge effort has been made recently to provide tools to optimally integrate due date assignment and scheduling decisions. In most cases it is common to assume that the assigned due date(s) are not restricted. However, in many practical cases, assigning due dates too far into the future may violate early agreements between the manufacturer and his customers. Thus, in this paper we extend the current literature to deal with such a constraint. This is done by analyzing a model that integrates due date assignment and scheduling decisions where each job may be assigned a different due date whose value cannot exceed a predefined threshold. The objective is to minimize the total weighted earliness, tardiness and due date assignment penalties. We show that the problem is equivalent to a two stepwise weighted tardiness problem, and thus for a large set of special cases it is strongly $\mathcal{NP}$-hard, even when the scheduling is done on a single machine. We then provide several special cases that can be solved in polynomial time, and present approximation results for a slightly modified (and equivalent) problem on various machine settings.



## 1. Introduction

Objective functions that include both earliness and tardiness penalties are now very popular in the scheduling literature due to the increasing interest in Just-in-Time (*JIT*) production in industry. When due dates are given in advance the most common objective in a *JIT* scheduling environment is to find a schedule $S$ that minimizes

$$Z(S) = \sum_{j=1}^{n} \left(\beta_j E_j + \gamma_j T_j\right), \tag{1}$$

where $J = \{1, \ldots, n\}$ is the set of $n$ jobs to be scheduled, and for any job $j \in J$, $d_j$ is the due date, and $\beta_j$ and $\gamma_j$ are nonnegative parameters representing the per unit earliness and tardiness costs, respectively. Moreover, for a given schedule $S$, $C_j$ represents the completion time of job $j$; $E_j = \max\left\{0, d_j - C_j\right\}$ represents the earliness of job $j$; and $T_j = \max\left\{0, C_j - d_j\right\}$ represents the tardiness of job $j$. Scheduling problems with the objective of minimizing $Z(S)$ in (1) are usually referred to as *earliness-tardiness scheduling problems*. Such problems arise in a scheduling environment where customers are not interested in receiving their jobs either earlier or later than the due date. Thus, if a job is finished prior its due date, it has to be held in inventory until that date. Consequently, this job incurs an earliness cost depending on the length of time it is held in inventory. This earliness penalty cost can result from deterioration, storage, insurance, etc. On the other hand, if a job is delivered after the due date, it incurs a tardiness cost that depends on how tardy the job is. The tardiness penalty can result from customer dissatisfaction, contract fines, and exposure to potential loss of reputation (Chen, 1996).

Earliness-tardiness scheduling problems have attracted the attention of many researchers (see, e.g., Sundararaghavan and Ahmed, 1984; Hall and Posner, 1991; Hall, Kubiak, and Sethi, 1991; Hassin and Shani, 2005; Davis and Kanet, 1993; Sivrikaya-Serifoğlu and Ulusoy, 1999 and Sourd and Kedad-Sidhoum, 2008). A survey on those problems can be found in Baker and Scudder (1990) and Lauff and Werner (2004). In all these papers, the main assumption is that due date quotation and scheduling decisions are made separately. Thus, due dates are given in advance as predefined parameters. In an integrated (centralized) system, however, due dates are determined by taking into account the ability to meet them. This is why an increasingly large number of recent studies view due date assignment (*DDA*) as part of the scheduling process and show how the ability to control due dates can be a major factor in improving system performance (see Gordon, Proth, and Strusevich, 2004 and Kaminsky and Hochbaum, 2004 for extensive surveys on this subject).

* Tel.: +972 8 6461389; fax: +972 8 6472958.
*E-mail address:* dvirs@bgu.ac.il

http://dx.doi.org/10.1016/j.ejor.2015.12.043

In a *JIT* scheduling environment where both due date assignment and scheduling decisions are integrated, it is common to include a due date assignment penalty within the objective function. This penalty reflects the fact that promising delivery dates too far into the future may force a company to offer price discounts in order to retain its business. In fact, the most common objective in the scheduling literature involving *DDA* decisions (see, e.g., Seidmann, Panwalkar, and Smith, 1981; Panwalkar, Smith, and Seidmann, 1982; Chen, 1996; Shabtay and Steiner, 2006 and Shabtay and Steiner, 2008; Mosheiov and Yovel, 2006; Li, Ng, and Yuan, 2011 and Drobouchevitch and Sidney, 2012) is to find a schedule $S$ and a set of due dates $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ that minimizes

$$Z(S, \mathbf{d}) = \sum_{j=1}^{n} \left(\alpha_j \max\left\{0, d_j - A_j\right\} + \beta_j E_j + \gamma_j T_j\right), \tag{2}$$

where for job $j \in J$, $\alpha_j$ is a nonnegative parameter representing the per unit lead time cost, and $A_j$ represents the lead time that is considered to be acceptable (and thus there is no lead time cost if the due date is set to be less than or equal to $A_j$).

Several methods to assign due dates have been considered in the literature. The most commonly used ones are (i) the common *DDA* method (usually referred to as the *CON DDA* method), where all the jobs are assigned the same due date; (ii) the *slack* due date assignment method (usually referred to as the *SLK DDA* method), where the jobs are given an equal flow allowance that reflects an equal waiting time; and (iii) the *unrestricted* due date assignment method (usually referred to as the *DIF DDA* method), where each job can be assigned a different due date. A common assumption is that the assigned due dates (in the *DIF* method) or the common due date or slack values (in the *CON* and *SLK* methods) are not restricted. However, in many practical cases, assigning a due date too far into the future may not be acceptable or may even violate an earlier agreement between the manufacturer and its customers. Thus, in this paper we extend the literature devoted to the *DIF DDA* by including a limitation on the assigned due dates. We do this by analyzing a set of scheduling problems in which the objective is to find a schedule $S$ and a set of due dates $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ that minimizes the objective in (2), subject to the condition that

$$d_j \le \bar{d}_j \tag{3}$$

for $j = 1, \ldots, n$, where $\bar{d}_j$ represents an upper limit on the assigned due date for job $j$.

For ease of presentation, we use the classical $X|Y|Z$ three-field notation introduced by Graham, Lawler, Lenstra, and Rinnooy Kan (1979) when referring to each scheduling problem. The $X$ field describes the machine environment, with $X \in \{1, Pm, Qm, Rm, Fm, Jm, Om\}$, where $X = 1$ implies that the scheduling is done on a single machine; $X = Pm$ implies that the scheduling is done on a set of $m$ identical parallel machines; $X = Qm$ implies that the scheduling is done on a set of $m$ uniform machines working in parallel; $X = Rm$ implies that the scheduling is done on a set of $m$ unrelated machines working in parallel; and $X = Fm$, $X = Jm$ and $X = Om$ refer to flow-shop, job-shop and open-shop scheduling systems, respectively. In all the above cases, it is assumed that $m$ is fixed, i.e., that the number of machines in the shop is not part of the instance. If, on the other hand, the number of machines is part of the instance, we remove the "$m''$ from the $X$ field, such that, for example, $X = P$ indicates that the scheduling is done on a set of $m$ identical parallel machines, where $m$ is not fixed. The $Y$ field includes the set of job-processing characteristics and constraints. In our framework $Y = \{\bar{Y},\ DIF\ , d_j \le \bar{d}_j\}$, where $DIF$ implies that the due dates are assignable according to the *DIF DDA* method, and $d_j \le \bar{d}_j$ imply that the assigned due date for job $j$ cannot exceed $\bar{d}_j$. $\bar{Y}$ is the set of all other job-processing characteristics and constraints (excluding $DIF$ and $d_j \le \bar{d}_j$). This set ($\bar{Y}$) may include entries such as $r_j$ which means that jobs may not be available from time zero; *pmtn* which means that preemption is allowed; *prec* which means that there are precedence constraints between jobs; and sp-graph which means that there is a precedence constraint relation that can be represented by a series-parallel graph. The $Z$ field contains the objective function for the scheduling problem, and it will usually refer to the one defined in (2).

Seidmann et al. (1981) were the first to study scheduling problems using the *DIF DDA* method. They focus on the case where (i) the scheduling is done on a single machine; (ii) the per unit earliness, tardiness and due date assignment costs are job-independent (that is $\alpha_j = \alpha$, $\beta_j = \beta$ and $\gamma_j = \gamma$ for $j = 1, \ldots, n$); (iii) the acceptable lead times are job-independent (that is $A_j = A$ for $j = 1, \ldots, n$); and (iv) there is no limitations on the value of the assigned due dates. They showed that the resulting $1|DIF|\sum_{j=1}^{n}(\alpha \max\{0, d_j - A\} + \beta E_j + \gamma T_j)$ problem is solvable in $O(n\log n)$ time. Shabtay and Steiner (2008) extended the analysis in Seidmann et al. (1981) to capture various multi-machine settings as well. They showed that for *any* machine environment $X$, the $X|DIF|\sum_{j=1}^{n}(\alpha \max\{0, d_j - A\} + \beta E_j + \gamma T_j)$ problem is equivalent to the $X\left|d_j = A\right|w\sum_{j=1}^{n} T_j$ problem, where $A$ is a fixed common due date for all jobs and $w = \min\{\alpha, \gamma\}$. Based on this equivalence they concluded that (*i*) problems $Pm|DIF|\sum_{j=1}^{n}(\alpha \max\{0, d_j - A\} + \beta E_j + \gamma T_j)$ and $Qm|DIF|\sum_{j=1}^{n}(\alpha \max\{0, d_j - A\} + \beta E_j + \gamma T_j)$ are ordinary $\mathcal{NP}$-hard; (*ii*) the $Pm|DIF|\sum_{j=1}^{n}\left(\alpha d_j + \beta E_j + \gamma T_j\right)$ problem is solvable in $O(n\log n)$ time; (*iii*) the $Rm|DIF|\sum_{j=1}^{n}\left(\alpha d_j + \beta E_j + \gamma T_j\right)$ is solvable in $O(n^3)$ time; and that (*iv*) problems $F2|DIF|\sum_{j=1}^{n}(\alpha d_j + \beta E_j + \gamma T_j)$ and $O2|DIF|\sum_{j=1}^{n}(\alpha d_j + \beta E_j + \gamma T_j)$ are strongly $\mathcal{NP}$-hard. Note, however, that based on the equivalence between the $X|DIF|\sum_{j=1}^{n}\left(\alpha \max\left\{0, d_j - A\right\} + \beta E_j + \gamma T_j\right)$ and $X\left|d_j = A\right|w\sum_{j=1}^{n} T_j$ problems ($w = \min\{\alpha, \gamma\}$), when either $\alpha = 0$ or $\gamma = 0$, we have that the objective value is equal to zero independent of the job schedule. The reason is that when $\alpha = 0$, by setting $d_j = C_j$ for $j = 1, \ldots, n$ we can obtain a solution with a zero value, which is independent of the actual $C_j$ values (i.e., independent of the job schedule). Similarly, when $\gamma = 0$ we can obtain a solution with a zero objective value by setting $d_j = 0$ for $j = 1, \ldots, n$. Here as well the result is independent of the actual schedule.

Shabtay and Steiner (2006) studied the extended version of the single machine problem, where the per unit earliness, tardiness and due date assignment costs are job-dependent as is the acceptable lead time. They proved that the resulting $1|DIF|\sum_{j=1}^{n}\left(\alpha_j \max\left\{0, d_j - A_j\right\} + \beta_j E_j + \gamma_j T_j\right)$ problem is equivalent to the well-known $1||\sum_{j=1}^{n} w_j T_j$ problem with $w_j = \min\left\{\alpha_j, \gamma_j\right\}$ and fixed due dates $d_j = A_j$ for $j = 1, \ldots, n$. Based on this and on the strongly $\mathcal{NP}$-hardness result for the $1||\sum_{j=1}^{n} w_j T_j$ problem (see Lawler, 1977), they concluded that the $1|DIF|\sum_{j=1}^{n}\left(\alpha_j \max\left\{0, d_j - A_j\right\} + \beta_j E_j + \gamma_j T_j\right)$ problem is strongly $\mathcal{NP}$-hard. Moreover, they showed that the $1|DIF|\sum_{j=1}^{n}\left(\alpha_j \max\left\{0, d_j - A_j\right\} + \beta_j E_j + \gamma_j T_j\right)$ problem is solvable in $O(n\log n)$ time when either $A_j = 0$ or when $A_j = A$ and $w_j = \min\left\{\alpha_j, \gamma_j\right\} = w$ for $j = 1, \ldots, n$.

Our main objective in this paper is to analyze the $X\left|\bar{Y}, DIF, d_j \le \bar{d}_j\right|\sum_{j=1}^{n}\left(\alpha_j \max\left\{0, d_j - A_j\right\} + \beta_j E_j + \gamma_j T_j\right)$ problem for various settings of $X$ and $\bar{Y}$. Owing to the results in Shabtay and Steiner (2006) above this problem is strongly $\mathcal{NP}$-hard even when $X = 1$, $\bar{Y} = \emptyset$ and $\bar{d}_j \ge \Sigma_{j=1}^{n} p_j$. Thus, we are mainly interested in exploring the borderline between easy and hard special cases of the problem, and providing approximation results for the hard cases.