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Decision Support

Multiple Criteria Hierarchy Process for ELECTRE Tri methods

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ABSTRACT

Sorting methods, in particular ELECTRE Tri methods, are widely used in Multiple Criteria Decision Aiding to deal with ordinal classification problems. Problems of this kind encountered in practice involve the evaluation of different alternatives (actions) on several evaluation criteria that are structured in a hierarchical way. In order to deal with a hierarchical structure of criteria in decision problems, Multiple Criteria Hierarchy Process (MCHP) has been recently proposed. In this paper, we apply the MCHP to the ELECTRE-Tri methods. In particular, we extend ELECTRE Tri-B, ELECTRE Tri-C and ELECTRE Tri-nC methods. We also adapt the MCHP concept to the case where interaction among evaluation criteria has either strengthening, or weakening, or antagonistic effect. Finally, we present an extension of the SRF method to determine the weights of criteria in case they are hierarchically structured.

1. Introduction

Multiple Criteria Decision Aiding (MCDA) methods deal with three major types of problems: choice, ranking and sorting (for a collection of surveys on MCDA see Figueira, Greco, and Ehrgott (2005)). Given a finite set of alternatives, choice problems consist in choosing a subset of best alternatives among the considered ones; ranking problems consist in rank ordering all the considered alternatives from the best to the worst, while sorting problems consist in assigning each alternative to one of the predefined and ordered categories (decision classes).

Among the best known sorting methods, there are the ELECTRE Tri methods and, in particular, the ELECTRE Tri-B (Yu, 1992), the ELECTRE Tri-C (Almeida Dias, Figueira, & Roy, 2010) and the ELECTRE Tri-nC (Almeida Dias, Figueira, & Roy, 2012) methods (for other sorting methods see Moscarola and Roy (1977); Nemery and Lamboray (2008)).

Given a set of alternatives $A=\{a_1,\ldots,a_n\}$ evaluated with respect to a set of evaluation criteria $\mathcal{G}$, and p categories $Cl_1,\ldots,Cl_p$ ordered from the worst to the best (such that $Cl_{h+1}\succ Cl_h$, for all $h=1,\ldots,p-1$), the ELECTRE Tri methods aim at assigning each alternative of A to one or more contiguous categories. In ELECTRE Tri-B, each category Cl_h is delimited by two reference profiles b_{h-1} and b_h. The reference profiles b_{h-1} and b_h are fictitious alternatives evaluated on all criteria from set $\mathcal{G}$. They are limiting category C_h from the bottom and the top, respectively. In ELECTRE Tri-C and ELECTRE Tri-nC methods, each category is characterized by one reference profile or more than one reference profile, respectively which correspond to some typical objects of this category. In all three methods, the assignment of alternatives to categories depends on the comparison of these alternatives with the reference profiles corresponding to the categories, the comparison that takes into account the whole set of criteria. It is worth mentioning that the assignment of one alternative is not influenced by the assignment of another alternative. Recently, Bouyssou and Marchant investigated in Bouyssou and Marchant (2015) the relationship between ELECTRE Tri-B and ELECTRE Tri-C, providing also another variant of ELECTRE Tri-B. On one hand, they demonstrated that some sorting assignments obtained by ELECTRE Tri-B cannot be obtained by fixing adequate profiles in ELECTRE Tri-C and viceversa; on the other hand, the authors adapted ELECTRE Tri-B so that the pessimistic and the optimistic recommendations can be obtained one from the other by a transposition operation.[1]

In all known sorting methods, all evaluation criteria are considered at the same level. However, practical decision problems often require consideration of a hierarchical structure of the set of

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[1] According to Bouyssou and Marchant (2015), the *transposition operation* consists in inverting the direction of preference on all criteria and in inverting the ordering of the categories.

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criteria. Considering a hierarchy of criteria is also a way of decomposing complex decision problems involving criteria referring to various levels of detail in the evaluation. For example, let us consider a council that evaluates a set of projects with respect to different aspects: economic, environmental, social, etc. In order to make these evaluations, the council has to consider several indicators for each macro criterion, and each indicator is composed, in turn, of other subcriteria, and so on. Even if global evaluation of a single project is well appreciated, members of the council would appreciate having an insight into partial evaluation of the projects, referring to a greater level of detail, e.g., with respect to economic, environmental or social aspects considered separately. The Multiple Criteria Hierarchy Process (MCHP) has been recently proposed for this purpose. It deals with decision problems where criteria are structured hierarchically, and at each level of the hierarchy, they are aggregated using one of available aggregation methods: either those involving additive (Corrente, Greco, & Słowiński, 2012) or non-additive (Angilella, Corrente, Greco, & Słowiński, 2015) value functions, or those involving outranking relations (Corrente, Greco, & Słowiński, 2013b).

The three main novelties introduced in this paper are the following:

- MCHP has been applied to ELECTRE Tri-B, ELECTRE Tri-C and ELECTRE Tri-nC in a way which permits to sort the alternatives into different categories not only in the root, at the comprehensive level, but also with respect to a lower level criterion, represented by an intermediate node of the hierarchy tree. In the ELECTRE framework, we take into account three different outranking relations based on the concordance and non-discordance tests; we specify two coherence properties that should hold for outranking methods applied to problems involving criteria organized in a hierarchical way, and we provide necessary and sufficient conditions on the cutting of the credibility of outranking, such that these two properties are satisfied. Moreover, some theorems show the relationship between the assignments done by ELECTRE Tri-B at different levels of the hierarchy, using both the pessimistic and the optimistic assignment procedures. We show that in case of a flat structure of criteria, the proposed methods boil down to the classical ELECTRE Tri methods and, for this reason, they can be considered their generalizations;
- The Simos–Roy–Figueira (SRF) method (Figueira & Roy, 2002), used to determine the weights of criteria in the classical ELECTRE methods, has been extended to handle criteria organized in a hierarchical way. In this case, following a top-down procedure, the weights of all criteria in the hierarchy, starting from the root criterion to the elementary criteria, are obtained. Even if the extension of the SRF method has been proposed in this paper to deal with sorting problems, it can be applied to choice and ranking problems involving criteria organized in a hierarchical way if these problems are solved using methods in which weights are interpreted as in ELECTRE methods, for example, when using PROMETHEE methods (Brans & Vincke, 1985);
- A methodology for dealing with different types of interactions between criteria organized in a hierarchical way has been proposed; extending the proposal of Figueira, Greco, and Roy (2009), in this paper we explain how to deal with decision making problems in which criteria organized in a hierarchy can present different types of interactions, such as synergy, redundancy, and antagonistic effects. Also in this case, even if this methodology has been proposed for dealing with sorting problems, it can be applied to choice and ranking problems approached by methods in which importance of criteria is interpreted as in ELECTRE methods.

The paper is organized as follows: Section 2 recalls the basic concepts of MCHP and ELECTRE Tri methods; in Section 3, we specify some coherence properties that should hold for hierarchical multiple criteria ELECTRE methods, as well as theorems ensuring them; in Section 4, we describe the application of MCHP to ELECTRE Tri methods, as well as their extension to the case of interacting criteria; an example of application of the proposed methodology to a real world decision making problem is presented in Section 5, while some concluding remarks and further directions of research are pointed out in Section 6.

2. MCHP and a brief reminder of ELECTRE Tri methods

In MCHP (Corrente et al., 2012), we consider a set of criteria structured in a hierarchical way, i.e., criteria are not considered at the same level, but they are distributed over l different levels; $\mathcal{G}$ denotes the entire set of criteria considered at all levels; $\mathcal{I}_{\mathcal{G}}$ is the set of indices of particular criteria representing positions of criteria in the hierarchy; $G_{\mathbf{r}}$ is a generic symbol of criterion from any level of the hierarchy; $n(\mathbf{r})$ is the number of subcriteria of $G_{\mathbf{r}}$ in the subsequent level, i.e., the direct subcriteria of $G_{\mathbf{r}}$ are $G_{(\mathbf{r},1)},\ldots,G_{(\mathbf{r},n(\mathbf{r}))}$; $g_{\mathbf{t}}:A\rightarrow\mathbb{R}$ denotes an elementary criterion, i.e., a criterion at level l of the hierarchy tree; EL is the set of indices of all elementary criteria; $E(G_{\mathbf{r}})$ is the set of indices of elementary criteria descending from $G_{\mathbf{r}}$; LBO is the set of indices of all subcriteria located at the last but one level of the hierarchy and $LB(G_{\mathbf{r}})$ is the set of indices of the subcriteria descending from $G_{\mathbf{r}}$ located at the last but one level of the hierarchy (for a more detailed description of the notation used in MCHP, see Corrente et al. (2012)).

Let us mention that, in the following, we shall suppose without loss of generality that all elementary criteria have an increasing direction of preference (the greater the evaluation of an alternative on an elementary criterion, the better the alternative is), and that when $\mathbf{r}=0$, then by $G_{\mathbf{r}}=G_{\mathbf{0}}$ we mean the entire set of criteria and not a particular criterion or subcriterion; in this particular case, $E(G_{\mathbf{0}})=EL$ and $LB(G_{\mathbf{0}})=LBO$.

For each elementary criterion $g_{\mathbf{t}}$, $\mathbf{t}\in EL$, the real number $w_{\mathbf{t}}$ represents a relative importance (weight) of $g_{\mathbf{t}}$ within the family of all elementary criteria, and we suppose, without loss of generality, that $\sum_{\mathbf{t}\in EL}w_{\mathbf{t}}=1$. The indifference, preference and veto thresholds for each elementary criterion $g_{\mathbf{t}}$ are denoted by $q_{\mathbf{t}}$, $p_{\mathbf{t}}$, and $v_{\mathbf{t}}$, respectively. $q_{\mathbf{t}}$ is the greatest difference between evaluations of alternative a and b on elementary criterion $g_{\mathbf{t}}$ compatible with the indifference among them; $p_{\mathbf{t}}$ is the smallest difference between evaluations of a and b on $g_{\mathbf{t}}$, being compatible with the preference of one alternative over the other; $v_{\mathbf{t}}$ is an upper bound beyond which the discordance about the outranking of one alternative over the other cannot surpass. For consistency, $0\le q_{\mathbf{t}}\le p_{\mathbf{t}}<v_{\mathbf{t}}$. In the following, for the sake of simplicity, we shall consider constant thresholds only, however, this assumption can easily be relaxed.

Note 2.1. Let us stress that, as we shall explain in Section 4, in ELECTRE Tri methods the assignment of an alternative to one of the considered categories depends on the comparison of the alternative with the reference profiles separating the categories. As the reference profiles can be seen as fictitious alternatives, the indices that we shall define in the following lines, are valid also for the comparison between two alternatives. For this reason, in defining these indices, by a and b we mean two alternatives or, an alternative and a reference profile.

In all considered adaptations of ELECTRE Tri methods to MCHP, we shall define the following indices: