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Decision Support

Linking validation: A search for coherency within the Supermatrix

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ABSTRACT

The consistency check within each pairwise comparison matrix is an important step in an Analytic Network Process (ANP) decision. In an ANP network there is both the ability and the need to test for additional levels of consistency or coherency among the priority vectors. Examples are used to highlight cases where a Supermatrix with priority vectors that were obtained from either perfect or nearly perfect consistent pairwise comparison matrices generates suboptimal decisions. Simulations are used to further demonstrate the frequency of these occurrences in general ANP networks. A form of cross validation within the Supermatrix called linking validation is developed and demonstrated. The linking validation method allows decision makers to use the priority vectors within the Supermatrix to validate other priority vectors within the Supermatrix. The linking validation method involves generating linking estimates. The linking estimates are compared against each other to identify the most incoherent priority vector by calculating the Linking Coherency Index (LCI) scores. The decision maker can then update the specified priority vector and repeat this process until the LCI-score for every linking estimate is below the given threshold. The use of linking validation to test for coherency further improves the validity of ANP models.

1. Introduction

The Analytic Network Process (ANP) is an amazing decision making method that takes advantage of our natural ability to make relative comparisons. The influential criteria in a decision can be organized into clusters within a network and through the aggregation of the relative comparisons the overall relative influence of the alternatives can be calculated. The ANP and a specific subset of ANP problems referred to as the Analytic Hierarchy Process (AHP) can be considered a disruptive technology; and since its initial debut it has been applied in many areas (Saaty & Ozdemir, 2008). As with any great tool or method the results are only as good as the user input; and hence the common phrase, “garbage in, garbage out,” can be used to emphasize the need for unbiased, reliable, and valid data under any method or approach including the ANP. One way to test and control for the quality of the inputs in an ANP model is the consistency index (Saaty, 1977) for the pairwise comparison matrix (PCM). Many variations of this method that focus on the PCM have been proposed (Ergu, Kou, Peng, & Shi, 2011).

Human decision makers are subject to all sorts of biases (Kahneman, 2011). While there is no reason to suspect the ANP exacerbates any of these biases, it would not be safe to assume the method serves as a total immunization from them. With the original consistency index, when one was not ordinally consistent and said that $a > b$, and $b > c$, but then said $c > a$ or if the intensity of the measured relationships was not consistent enough, it would not be advisable to proceed because of data quality issues within the PCM. More about the consistency index in a PCM is discussed in greater detail in the literature review; but what about consistency beyond the PCM?

As we think of realms of influence in an ANP network, the priority vectors obtained from the PCMs are aggregated within a much larger realm of influence within an ANP Supermatrix. As a Supermatrix is an aggregate of multiple measurements across many scales, it is difficult to see how one could check for a greater level of consistency beyond the PCM. But what if one could? What if there was a way to cross validate the overall Supermatrix for a level of consistency or coherency that would serve as a data quality check for the priority vectors within the Supermatrix? One might also wonder why such a test would be needed if there is a high level of, or even perfect, consistency within the PCMs. After reviewing the relevant literature, Section 3 motivates the need for such a test with examples to highlight realistic scenarios where a problematic Supermatrix can arise and would otherwise currently

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go undetected. While this new safeguard could be termed a Supermatrix consistency check, it is simply referred to herein as linking validation and coherency testing. Next, in Section 4, simulated decisions further demonstrate in general that performing coherency testing within a Supermatrix is crucial in order to obtain meaningful results with the ANP. In Section 5, a Linking Coherency Index (LCI) is proposed to provide another level of checks and safeguards to improve the reliability of ANP decision models. Section 6 contains a sample decision Supermatrix that is derived from the examples in Section 3 where the LCI is demonstrated to identify the problematic priority vectors. Finally, in Section 7, concluding remarks and suggestions for future research are discussed.

2. Literature review

First, the relevant literature regarding methods to check for consistency within a PCM is highlighted. Other reviews provide detailed critiques of the pros and cons of the different methods; and because this new approach is not at the level of the PCM such a critique is tangential and will not be addressed herein. Next, to understand the solution provided in Section 5 it is worthwhile to review some additional findings regarding linking pins and the units of measurement in priority vectors. These concepts will now be addressed in greater detail beginning with the check for consistency.

2.1. Consistency

The consistency issue within a PCM has been categorized into the following three categories (1) cardinal inconsistency, (2) ordinal inconsistency and (3) both (Kou, Ergu, & Shang, 2014). The first calculations of consistency within an AHP PCM were proposed by Saaty (Saaty, 1980; Vargas, 1982). The value of improving consistency within a PCM has been demonstrated through simulations (Herman & Koczkodaj, 1996; Siraj, Mikhailov, & Keane, 2012) and other examples from the literature that have proposed new approaches. Ozdemir (2005) calls for more redundancy in comparisons and shows how inconsistency measurements can be generalized to a collection of clusters. Multiple methods have been developed to address and reduce cardinal inconsistency within a PCM (Cao, Leung, & Law, 2008; Harker, 1987; Saaty, 1980; Zeshui & Cuiping, 1999). Similarly many algorithms have been developed to study and improve the ordinal consistency (Ali, Cook, & Kress, 1986; Gass, 1998; Kwiesielewicz & Van Uden, 2004; Mikhailov & Knowles, 2010; Pahikkala, Waegeman, Tsivtsivadze, Salakoski, & De Baets, 2010). There has also been research to address and resolve both forms of inconsistency within a PCM (Genest & Zhang, 1996; González-Pachón & Romero, 2004; Kou et al., 2014; Li & Ma, 2008). Each of these methods focuses on consistency at the level of the PCM and can be employed to improve consistency at that level. Additionally, some methods have been proposed specifically for applications within the ANP (Ergu et al., 2011; Leung, Hui, & Zheng, 2003). All of these methods are useful; and consistency at the PCM level must be addressed in order to develop meaningful decision models. There are also some additional relationships and links among the priority vectors within a Supermatrix that should be discussed because of the important role they will play in the ability to validate the priority vectors within a Supermatrix.

2.2. Linking and units

Saaty (2005) suggests using a form of linking or "pivot" comparisons among non-homogeneous items where the relationships between the object's sizes exceed the use of the 1–9 scale. This example was extended by Saaty and Shang (2011) to demonstrate how acts of kindness or service, termed "intangible contributions to society" can be measured and compared by using pivot comparisons. Schoner, Wedley, and Choo (1993) propose a linking pin approach in an effort to address and unify approaches to the AHP. The linking pin approach is a normalization process that can be used in both a hierarchy and a network. They show linking can be done because the criteria and alternatives are structurally dependent on one another. From this dependency it can be seen that everything in the Supermatrix is related. The importance of this dependence and the ability to link will be highlighted again and demonstrated in Section 5.

The concept of structural dependence is easy to see within a given criterion cluster. One issue that may arise when combining separate criteria clusters or networks is the question about what then is the unit of measurement in the overall network? Criteria weights in general are misunderstood and misused (Choo, Schoner, & Wedley, 1999). Choo et al. (1999) demonstrate that there is no consensus on the meaning or manner of deriving criteria weights. Furthermore, the criteria weights should not be calculated in a way that is independent of how they are used in decision model. While criteria weights can be used for the column normalization process, normalization in and of itself does not remove the units from the criteria being considered. Wedley and Choo (2001), explain that ratio scales in the ANP have a unit of measure and the unit of measure is important and useful. The unit of measurement is derived from the topmost node. The scale that one can obtain from such a unit is transient depending on the alternatives being considered but so is the ratio scale itself. Focusing on the ratios rather than the rank will improve the efficacy of the ANP. Wedley and Choo (2011) conclude "Therein lie both the advantage and dilemma of AHP. We do not need explicit knowledge of the underlying unit of measure to derive a ratio scale, yet the derived scale has a unit." This understanding that the unit of measurement is derived from the topmost node provides a unit to use as the basis for comparing criteria across clusters. A novel application of linking pin comparisons and understanding the units of measurement will allow the creation of additional estimates to cross validate the data in the Supermatrix.

3. Examples

We begin with a hypothetical scenario to conceptually motivate the need for a coherency test to check for this higher level of consistency. The term coherency is borrowed from Bayesian analysis (Hastings & Gross, 2012) and metrology (VIM, 2004). Coherent data can be defined as self-consistent and non-contradictory with respect to a particular system. The coherency test is fully developed after the frequency of this issue occurring in general ANP models is shown in three examples within this section and then generalized in the simulations conducted in Section 4.

Assume a decision maker is trying to choose between which of two individuals to ask on a date; and decides to evaluate them based on the following two intangible criteria: intelligence and attractiveness. Furthermore, assume the eligible individuals are both equally attractive and equally intelligent; and also that both criteria equally contribute to each of the persons' overall value. In other words, the priority vector for Person 1 and Person 2 would be (.5, .5). While in practice this model with two equally preferred alternatives wouldn't be very a useful model it will highlight the limitations of only checking for consistency within the PCM. As the decision maker makes the pairwise comparisons they will all be perfectly consistent because each PCM only has two elements. Sample priority vectors are provided below where we also assume that when comparing the two individuals against each other with respect to intelligence an incorrect yet perfectly consistent comparison was made.