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# The Vehicle Routing Problem with Simultaneous Pick-ups and Deliveries and Two-Dimensional Loading Constraints

Emmanouil E. Zachariadis[a,*], Christos D. Tarantilis[a], Chris T. Kiranoudis[b]

[a] Department of Management Science & Technology, Athens University of Economics and Business, Greece
[b] School of Chemical Engineering, National Technical University of Athens, Greece

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ABSTRACT

We introduce and solve the Vehicle Routing Problem with Simultaneous Pick-ups and Deliveries and Two-Dimensional Loading Constraints (2L-SPD). The 2L-SPD model covers cases where customers raise delivery and pick-up requests for transporting non-stackable rectangular items. 2L-SPD belongs to the class of composite routing-packing optimization problems. However, it is the first such problem to consider bi-directional material flows dictated in practice by reverse logistics policies. The aspect of simultaneously satisfying deliveries and pick-ups has a major impact on the underlying loading constraints: feasible loading patterns must be identified for every arc traveled in the routing plan. This implies that 2L-SPD generalizes previous routing problem variants with two-dimensional loading constraints which call for one feasible loading per route. From a managerial perspective, the simultaneous service of deliveries and pick-ups may bring substantial cost-savings, but the generalized loading constraints are very hard to tackle in reasonable computational times. To this end, we propose an optimization framework which employs memorization techniques designed for the 2L-SPD model, to accelerate the solution methodology. To assess the performance of our routing and packing algorithmic components, we have solved the Vehicle Routing Problem with Simultaneous Pick-ups and Deliveries (VRPSPD) and the Vehicle Routing Problem with Two-Dimensional Constraints (2L-CVRP). Computational results are also reported on newly constructed 2L-SPD benchmark problems. Apart from the basic 2L-SPD version, we introduce the 2L-SPD with LIFO constraints which prohibit item rearrangements along the routes. Computational experiments are conducted to understand the impact of the LIFO constraints on the routing plans obtained.



## 1. Introduction

In the last years, advances both in optimization methodologies and computer systems allow researchers and practitioners to examine practical optimization problems which in the past were thought to be too complex to handle. One such research stream that has emerged in the logistics optimization literature is devoted to the analysis of problems which are aimed at effectively dispatching a fleet of vehicles and at the same time, ensuring that the transported items can be feasibly loaded into these vehicles. The problem introduced in the present study belongs to this class of integrated vehicle routing and loading problems. Briefly, the presented model, referred to as *the Vehicle Routing Problem with Simultaneous Pick-Ups and Deliveries and Two Dimensional Loading Constraints* (2L-SPD) calls for the generation of the optimal routes to fully satisfy the demand raised by a set of customers. The demand of each customer consists of two transportation requests: the first one is associated with a set of items that must be transported from the warehouse to the customer location, whereas the second request is associated with a set of items that must be transported from the customer location to the central warehouse. The items that are transported from and to the warehouse are considered rectangular and not stackable. Thus, 2L-SPD is aimed at generating feasible, two-dimensional, orthogonal loading patterns for the transported items carried by the produced route set.

The main innovative feature of the examined model, compared to previously introduced vehicle routing variants with loading constraints, lies in the fact that vehicles offer simultaneous pick-up and delivery service. This implies that the item sets carried along a route change drastically: delivery items are unloaded and additional pick-up items are loaded onto the vehicle. Thus, the loading feasibility must be examined for every arc traveled by the routes. On the contrary, previously introduced delivery models assume that the size of the item set carried by a vehicle monotonically decreases, so that the loading feasibility has to be tested, only when the corresponding vehicle leaves the warehouse fully loaded. As a result, previously

* Corresponding author. Tel.: +30 6945028191; fax: +30 210-8816705.
*E-mail address:* ezach@aueb.gr, ezach@mail.ntua.gr, mzaxariadis@gmail.com (E.E. Zachariadis).

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examined delivery models with two-dimensional loading constraints can be regarded as a special case of 2L-SPD, when for all customers, the pick-up requests are set to an empty item set.

Regarding the routing characteristics, 2L-SPD is a generalization of the Vehicle Routing Problem with Simultaneous Pick-ups and Deliveries (VRPSPD) which calls for the optimal routes that simultaneously offer pick-up and delivery service, under one-dimensional loading constraints (Dell'Amico, Righini, & Salani, 2006; Subramanian, Uchoa, Pessoa, & Ochi, 2013a). Analogously, VRPSPD is a generalization of the basic version of the vehicle routing problem (VRP) which is aimed at producing the optimal delivery route set subject to one-dimensional capacity constraints.

As already stated, 2L-SPD belongs to the integrated vehicle routing and multi-dimensional packing problems which jointly call for the optimal route planning and feasible packing structures for the transported goods. The first such problem has been introduced by Iori, Salazar-Gonzãlez, and Vigo (2007) who examine a vehicle routing extension with two-dimensional loading constraints: vehicles are considered to deliver rectangular items (boxes, pallets) which are not stackable. This problem is referred to as the vehicle routing problem with two-dimensional loading constraints (2L-CVRP). Under 2L-CVRP, the minimum cost set of routes must be generated for the vehicle fleet. For each of these routes, a feasible orthogonal two-dimensional packing must be determined for the transported items. The authors present a branch-and-cut method for dealing with small-scale problems (up to 25 customers and 91 boxes). To solve larger-scale instances, researchers have proposed various metaheuristic solution strategies: A tabu search methodology has been developed by Gendreau, Iori, Laporte, and Martello (2008). Zachariadis, Tarantilis, and Kiranoudis (2009) have proposed a tabu search and guided local search hybridization for the routing aspects and a bundle of packing heuristics for the loading requirements. Fuellerer, Doerner, Hartl, and Iori (2009) have developed an ant colony optimization approach. Another tabu search-guided local search hybrid has been proposed by Leung, Zhou, Zhang, and Zheng (2011). Strodl, Doerner, Tricoire, and Hartl (2010) have proposed a 2L-CVRP solution method emphasizing on the development of efficient data structures for storing obtained loading feasibility information. More recently, Duhamel, Lacomme, Quilliot, and Toussaint (2011) have solved the 2L-CVRP by a greedy randomized adaptive search (GRASP) and evolutionary local search (ELS) solution approach, while Zachariadis, Tarantilis, and Kiranoudis (2013) have proposed a simple-structured local search methodology. The most recent works on the 2L-CVRP are due to Dominguez, Juan, and Faulin (2014) and Wei, Zhang, Zhang, and Lim (2015) who introduce a Variable Neighborhood Search method. An additional routing model with two-dimensional loading constraints has been introduced by Malapert, Guéret, Jussien, Langevin, and Rousseau (2008). The authors present a pick-up and delivery model which assumes that non-stackable rectangular items have to be transported between pairs of service locations. An additional class of integrated routing-packing models considers three-dimensional loading constraints. This model category is applicable for logistics applications where the transported boxes can be stacked one on top of the other. The first such study is due to Gendreau, Iori, Laporte, and Martello (2006). Their work introduces the vehicle routing problem with three-dimensional loading constraints (3L-CVRP) which generalizes 2L-CVRP by calling for feasible, three-dimensional loading arrangements. Additional requirements met in practice are examined: fragility constraints, stability rules for the transported cargo and easy unloading operations. Several metaheuristic developments have been proposed for the 3L-CVRP (Bortfeldt, 2012; Fuellerer, Doerner, Hartl, & Iori, 2010; Ruan, Zhang, Miao, & Shen, 2011; Tarantilis, Zachariadis, & Kiranoudis, 2009; Zhu, Qin, Lim, & Wang, 2012). A relevant model is due to Männel and Bortfeldt (2013). The latter work introduces a pickup and delivery problem where three-dimensional and stackable items are transported between customer locations. For a detailed list of vehicle routing models which explicitly deal with loading constraints, the interested reader is referred to the reviews of Iori and Martello (2010), Iori et al. (2013) and Perboli, Gobbato, and Perfetti (2014).

The purpose of the present paper is to formally introduce the 2L-SPD model. An efficient solution approach is developed and presented for the 2L-SPD. The proposed solution approach consists of two algorithmic components: one for the routing and one for the packing aspects. Both components are based on our algorithm presented for the 2L-CVRP (Zachariadis et al., 2013). However, they are extended to provide higher-quality solutions. In addition, we present a new original framework which combines the individual routing and packing components for efficiently dealing with the special requirements of the 2L-SPD model. We also present feasibility memory structures that have been specially designed for the 2L-SPD and drastically accelerate the search process. The overall 2L-SPD solution approach is a robust optimization methodology efficiently dealing with instances of hundreds of customers and items.

In addition to the basic 2L-SPD model, we introduce the 2L-SPD with LIFO constraints. Under the LIFO variant, item rearrangement along the routes is not allowed, so that the loading requirements become much tighter leading to lower area utilization.

To assess the effectiveness of both the routing and packing ingredients of our algorithm, computational results are reported on well studied VRPSPD and 2L-CVRP benchmark instances. Then, computational results are reported on newly constructed 2L-SPD test cases both for the basic 2L-SPD, as well as the LIFO constrained variant.

The remainder of the present paper is as follows: Section 2 presents in detail the examined problem and discusses its applicability for practical logistics operations. Section 3 describes the proposed 2L-SPD local search solution approach. This master local search algorithm makes use of two loading feasibility examination components which are described in Sections 4 and 5. Then, Section 6 provides the necessary methodological modifications for tackling the LIFO version of the 2L-SPD model. In Section 7, extensive computational results are reported for the VRPSPD, 2L-CVRP and 2L-SPD models. In addition, comparisons are made between the obtained results and the ones of previously published methodologies. Finally, Section 8 concludes the paper.

## 2. The 2L-SPD model

In this section, we present a formal description of the 2L-SPD model, followed by some 2L-SPD practical applications. Then, we introduce the 2L-SPD variant with LIFO constraints which prohibits rearrangement of items along the vehicle trips.

### 2.1. Description of the basic 2L-SPD model

Let $G = (V, A)$ be a complete graph where $V = \{0, 1,\dots, n\}$ is the vertex set and $A$ is the set of arcs $(i, j)$ connecting every pair of distinct vertices. Each arc $(i, j) \in A$ is associated with a cost $c_{ij}$ equal to the distance that must be traveled for moving from vertex $i$ to vertex $j$. Vertex 0 represents the warehouse which acts as the base station of $k$ homogeneous vehicles. Each vehicle has a maximum carrying weight equal to $Q$ and a loading surface of length and width equal to $L$ and $W$, respectively. Vertex set $N = V \backslash \{0\}$ corresponds to the customer set. With each customer $i \in N$, there are two associated item sets, namely $D_i$ and $P_i$. Set $D_i$ corresponds to the items that must be delivered from the warehouse to the customer, whereas $P_i$ contains the items that must be picked-up from customer $i$ and transported to the warehouse. All transported items are considered non-stackable. The total weight of item sets $D_i$ and $P_i$ are equal to $d_i$ and $p_i$, respectively. The length and width dimensions of an item $j \in D_i \cup P_i$, $(\forall\, i \in N)$ are denoted by $l_j$ and $w_j$, respectively.