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The bi-objective mixed capacitated general routing problem with different route balance criteria

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ABSTRACT

In the mixed capacitated general routing problem, one seeks to determine a minimum cost set of vehicle routes serving segments of a mixed network consisting of nodes, edges, and arcs. We study a bi-objective variant of the problem, in which, in addition to seeking a set of routes of low cost, one simultaneously seeks a set of routes in which the work load is balanced. Due to the conflict between the objectives, finding a solution that simultaneously optimizes both objectives is usually impossible. Thus, we seek to generate many or all efficient, or Pareto-optimal, solutions, i.e., solutions in which it is impossible to improve the value of one objective without deterioration in the value of the other objective. Route balance can be modeled in different ways, and a computational study using small benchmark instances of the mixed capacitated general routing problem demonstrates that the choice of route balance modeling has a significant impact on the number and diversity of Pareto-optimal solutions. The results of the computational study suggest that modeling route balance in terms of the difference between the longest and shortest route in a solution is a robust choice that performs well across a variety of instances.

1. Introduction

Routing problems arise in many practical applications and have been widely studied. Given a network, one or several routes need to be created visiting all, or a subset of, the network's nodes, arcs, and/or edges. An overview and early references can be found in Laporte and Osman (1995) and a history can be found in Laporte (2009). Two popular variants are the capacitated vehicle routing problem (CVRP) and the capacitated arc routing problem (CARP). In the CVRP, the nodes in the network need to be served, while in the CARP the arcs/edges need to be served.

We consider the mixed capacitated general routing problem (MCGRP), a generalization of both the CVRP and the CARP. It is defined on a mixed network $N = (V, E, A)$, where V is a set of nodes, E a set of undirected edges, and A a set of directed arcs. Some of the nodes, edges, and arcs are *required*, i.e., they have a non-negative demand that needs to be served. Each edge and each arc have a non-negative traversal cost, and required nodes, edges, and arcs may have a non-negative service cost. A fixed homogeneous fleet of vehicles with a given service capacity is available for serving the demand. Typical applications are waste collection and newspaper distribution where arcs/edges represent streets and nodes represent facilities, e.g., schools, and residential and commercial buildings.

The MCGRP was introduced by Pandi and Muralidharan (1995), who propose a heuristic solution procedure based on creating one giant route which is subsequently split into individual vehicle routes. The literature on MCGRP is scarce, but includes both heuristic procedures and exact solution approaches: Gutiérrez, Soler, and Hervás (2002) propose a heuristic based on the cluster-first route-second principle. Their heuristic is compared with the one by Pandi and Muralidharan (1995) and shows promising results. Prins and Bouchenoua (2005) develop a memetic algorithm and introduce a set of benchmark instances. Other methods explored are simulated annealing (Kokubugata, Moriyama, & Kawashima, 2007) and adaptive iterated local search (Dell'Amico, Díaz, Hasle, & Iori, 2015). The first integer programming formulation is given by Bosco, Laganà, Musmanno, and Vocaturo (2013) who use a branch-and-cut algorithm to find optimal solutions to small instances. Combinatorial bounds were studied by Bach, Hasle, and Wøhlk (2013). The MCGRP with

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stochastic demands is introduced by Beraldi, Bruni, Laganà, and Musmanno (2015). A chance-constrained integer programming formulation is proposed, and solved exactly by a branch-and-cut algorithm and heuristically by a neighborhood search algorithm.

Most previous studies of the MCGRP consider a single objective: minimize total route cost. We study a bi-objective version of the problem where, in addition to seeking to minimize total travel cost, we simultaneously seek to balance the route lengths. In this paper, we assume that cost is a linear function of the travel distance and that a set of routes is balanced if the travel distances of the routes are similar. This extension of the MCGRP is motivated by its practical relevance, because in many real-life applications, it is desirable for drivers to have comparable workloads. To the best of our knowledge, there is only one publication on the bi-objective MCGRP with route balance: Mandal, Pacciarelli, Løkketangen, and Hasle (2015) consider the bi-objective MCGRP where route balance is defined as the difference between the longest and the shortest route. A memetic algorithm is proposed as solution method. We note that route balance can be defined in various ways, and one of our goals is to investigate the impact of this modeling choice.

The main contributions of this paper are that we (1) provide new insights in the simultaneous minimization of route cost and route balance for vehicle routing problems, and (2) provide new insights in the effect of the choice of route balance modeling on the Pareto front. Our computational study, using small benchmark instances of the mixed capacitated general routing problem, demonstrates that the choice of route balance modeling has a significant impact on the Pareto front in terms of the number of Pareto-optimal solution and diversity of Pareto-optimal solutions. These insights may allow practitioners and academics to make more informed choices when including route balance as an objective in vehicle routing problems.

The remainder of this paper is organized as follows. Section 2 presents insights on the effects of considering both route cost and route balance in routing problems. Section 3 discusses related and relevant literature. Section 4 introduces a method for solving the bi-objective MCGRP. Section 5 presents the numerical results of a computational study. Finally, Section 6, contains concluding remarks and suggestions for future research.

2. Bi-objective vehicle routing problems—some insights

Bi-objective routing problems belong to the class of multi-objective optimization problems, see e.g., Ehrgott (2005). These problems can be described as follows:

$$\min_{x\in\mathcal{X}} z(x) := \{z_1(x), z_2(x), \ldots, z_p(x)\},$$

where $\mathcal{X} \in \mathbb{R}^n$ represents the feasible set in the decision space, and the image $\mathcal{Y}$ of $\mathcal{X}$ under vector-valued function $z = \{z_1, \ldots, z_p\}$ represents the feasible set in the criterion space, i.e., $\mathcal{Y} := z(\mathcal{X}) := \{y \in \mathbb{R}^p : y = z(x) \text{ for some } x \in \mathcal{X}\}$. Because there are multiple objectives, there will be, in most cases, multiple solutions that can be considered optimal, or non-dominated. Such solutions are often referred to as *Pareto optimal*. More specifically, we have:

Definition. A solution $x^* \in \mathcal{X}$ is a *weak* Pareto optimum if and only if there is no $x \in \mathcal{X}$ such that $z_i(x) < z_i(x^*)$ for $i = 1, \ldots, p$.

Definition. A solution $x^* \in \mathcal{X}$ is a *strict* Pareto optimum if and only if there is no $x \in X$ such that $z_i(x) \leq z_i(x^*)$ for $i = 1, \ldots, p$ and $z(x) \neq z(x^*)$.

The image of all strict Pareto optimal solutions is called the *Pareto front*, alternatively the efficient frontier or the nondominated frontier. In the bi-objective case, there are exactly two objectives $z_1(x)$ and $z_2(x)$, i.e., $p = 2$.

We will study the Pareto front of routing problems with two objectives: minimize the route cost and maximize the route balance. In this section, we use the CVRP to illustrate features of bi-objective routing problems. The insights are, however, general and also hold for the CARP and the MCGRP. We make the following assumptions:

1. There is a fleet of m homogeneous vehicles with capacity Q that has to serve n customers, each with a demand that is less than or equal to the vehicle capacity.
2. A solution consists of a single route for each vehicle, starting and ending at the depot and serving at least one customer. Furthermore, each customer is served by exactly one vehicle.

The first objective, which is also the typical objective in routing problems, is to minimize the total cost of the solution, i.e.,

$$\min \quad \sum_{r\in\Omega} t_r,$$

where Ω is the set of all possible routes and t_r is the cost of route r. For ease of presentation, we assume that the cost of a route is the travel distance of the route. The second objective is to maximize the route balance, or minimize the route imbalance, where we define route balance with respect to the travel distance of the routes in the solution.

The first challenge encountered when solving bi-objective routing problems with route balance as one of the objectives is choosing how to model route balance. Two natural choices are:

$$\min \quad \sum_{r\in\Omega} (t_r - \mu)^2,$$

and

$$\min \quad t^L - t^S,$$

where μ is the average travel distance of the routes in the solution, and t^L and t^S are the travel distance of the longest and shortest route in the solution, respectively. The first choice is equivalent to minimizing the variance of the travel distance of the routes in the solution.

Only for a perfectly route balanced solution will these two route balance objectives have the same optimal value: 0. Given that a perfectly route balanced solution rarely has minimum total travel distance, these two route balance objective will likely result in substantially different Pareto fronts. An important difference between the two route balance objectives is that the first explicitly takes the travel distance of all routes in the solution into account, whereas the second only implicitly takes the travel distance of all routes in the solution into account by ensuring that all travel distances are between the longest and shortest travel distance. If a decision maker seeks to create routes that result in an equitable workload for the drivers, i.e., a fair distribution for the travel distances of the routes, a variance objective may be more suitable. However, a variance objective may also result in solutions with extremes, i.e., one substantially longer or shorter route, which may be undesirable. Such extremes may be avoided by imposing lower and upper bounds on the individual route length.

Using a variance objective may also be more computationally demanding, because the mean length of routes in a solution is not known in advance and because variance is a quadratic function. Two alternative formulations where travel distances of all routes are considered are as follows:

$$\min \quad \sum_{r\in\Omega} \left(t_r - t^S\right),$$

and

$$\min \quad \sum_{r\in\Omega} |t_r - T^G|,$$

where t^S, as before, is the travel distance of the shortest route in the solution, and where t^G is a prespecified target route travel distance. For some real-life routing problems a natural target travel distance