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Decision Support

Selective majority additive ordered weighting averaging operator

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ABSTRACT

Usually, in order to summarize various opinions about a particular situation (mainly product or service valuation on Internet) a process called aggregation is used. This process basically consists of determining the appropriate value to represent the majority's opinion and many strategies and operators can be used for this purpose. Simple arithmetic mean is widely used to resume several opinions in a single value, but this value is generally not representative or it is affected by the extreme values. An alternative to aggregate opinions are the Ordered Weighting Averaging (OWA) operators. Nevertheless, they have distribution problems when applied to aggregates with cardinalities. These problems may be solved by using Majority Additive OWA (MA-OWA) operator, a sort of arithmetic mean of arithmetic means. MA-OWA operator works adequately but, in some cases, discards the minority's opinion, specifically when it does not coincide with the largest cardinality value. In order to generalize the usage of MA-OWA operator, the rest of opinions are taken into account using a Cardinality Relevance Factor. This paper introduces a Selective Majority Additive OWA (SMA-OWA) which manages the significance of all opinions varying the Cardinality Relevance Factor. Mathematical extension of SMA-OWA, its properties and some illustrative examples are presented in this article.

1. Introduction

Many situations of e-commerce, such as buying a book, selecting a video camera, booking a hotel room, etc., imply decisions based on information provided by other customers. Similarly, selecting candidates for a senior position, voting a law, rating a news item in a newspaper (with the reader's point of view), measuring popularity, deciding on the restructuring of local government, etc., are activities that make it desirable to have a consensus value from everyone's opinion. In general aggregation process, finding a suitable value to represent the majority's opinion is not an easy task. Frequently, individual opinions are summarized in a single value by using an arithmetic mean. Although for many cases this mechanism allows to obtain a good idea of quality or adequateness of certain product or measure the representativeness of different candidates, there are others whose aggregated values are not representative of the majority's opinion.

Aggregation appears in many applications related to the development of intelligent systems, such as multi-criteria decision making, fuzzy group decision making (Yan & Ma, 2015), fuzzy logic controllers or fuzzy systems modeling (Fodor & Roubens, 1994; Yager & Filev, 1994).

This reduction of individual values into a representative value or a consensual judgment or the majority's opinion (Pasi & Yager, 2003) is not trivial. In order to face the mentioned problem several operators such as weighted means, quasi-weighted means (Xu & Da, 2004), ordered weighted averaging (OWA) (Yager, 1988) and their families (Yager, 1993) have been proposed. However, OWA operators for generalizing majorities are conditioned to the use of α-cuts of dual strictly monotonic OWA operators (Llamazares, 2004).

Specifically, in multi-agent decision making it is often necessary to count on an overall opinion which synthesizes that of the majority of the decision makers. In the fuzzy approaches to multi-agent decision making, the concepts of consensus and majority are modeled using computing with words (Herrera, Alonso, Chiclana, & Herrera-Viedma, 2009; Peláez & Doña, 2003a), i.e., by means of linguistic quantifiers, linguistic terms (Massanet, Riera, Torrens, & Herrera-Viedma, 2014) and linguistic preferences (Alonso, Pérez, Cabrerizo, & Herrera-Viedma, 2013) fuzzy concepts referred to the quantity of elements of given reference sets (Kacprzyk, Nurmi, & Fedrizzi, 1997) operators that introduce individual and group quantification strategies (Peláez & Doña, 2006; Yager, 1996) and models for heterogeneous group decision making problems guided by the heterogeneity criterion (Pérez, Cabrerizo, Alonso, & Herrera-Viedma, 2014) and granulation of the linguistic terms (Cabrerizo, Herrera-Viedma, & Pedrycz, 2013).

Although OWA operators are widely used for aggregation, some of their analysis for modeling the majority concept show that usual

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definition of OWA operators based on linguistic quantifiers does not capture the semantics of a consensus (Peláez, Doña, & Gómez-Ruiz, 2007). To summarize the majority's opinion, several approaches have been proposed (Pasi & Yager, 2003; Peláez, 2001). For example, Yager and Filev (Yager & Filev, 1999) define the induced ordering weighted averaging (IOWA) operators, which obtain a scalar value for a majority's opinion, and Pasi and Yager (2003) use the vague concept of majority's opinion (fuzzy majority) as a fuzzy subset. Also, in Bustince, Jurio, Pradera, Mesiar, and Beliakov (2013) a generalization of the weighted voting method used in the exploitation phase of decision making problems with preference relations has been recently proposed.

Summarizing, the direct use of conventional OWA operators, repeating the elements with cardinality larger than unity, does not always produce reasonable results, and distribution problems, such as cake-cutting problems may appear (Peláez & Doña, 2003b). In fact, most common aggregation operators overemphasize the opinion of the minority at the expense of that of the majority. To solve this problem, Majority Additive OWA (MA-OWA) operators have been introduced (Peláez & Doña, 2003b). These operators use a weight vector which depends on the cardinalities of the aggregates, and can be interpreted as an arithmetic mean of arithmetic means. A linguistic aggregation extension of these operators has also been introduced in (Peláez & Doña, 2003a).

Even though MA-OWA operator obtains more accurate results than other operators, they present quick convergence to the value with the largest cardinality (Peláez & Doña, 2003b). I.e., when the cardinality of an element from the rest excessively grows, its weight tends to one and the other weights tend to zero. In order to face this issue, this paper introduces and describes the Selective Majority Additive OWA (SMA-OWA), which allows choosing the importance given to the largest cardinality. Section 2 briefly reviews the class of OWA and MA-OWA operators, stating the notation used for the rest of this manuscript. In Section 3, the formulation and some illustrative examples of SMA-OWA proposed operator are presented. In addition, two alternatives to determine the value of the Cardinality Relevance Factor are proposed. In Section 4, the use of SMA-OWA operator is validated by using: (a) the usual properties required for an aggregation operator, (b) the weights decomposition process used to explain the way they increase or decrease their values and (c) an exhaustive analysis over a real study case. Finally, conclusions and future works are summarized in Sections 5 and 6, respectively.

2. Preliminaries

Aggregation operators are special real functions with inputs from a subdomain $\mathbb{I}$ of the real line. The basic feature of all aggregation functions is their non-decreasing, monotonicity and boundary conditions. The increase of input values cannot decrease the output values and they are aggregated in the same scale of input values, respectively. Formally (Grabisch, Marichal, Mesiar, & Pap, 2011), an *aggregation function* in $\mathbb{I}^n$ is a function $F^{(n)} : \mathbb{I}^n \to \mathbb{I}$ that:

i. is non-decreasing (on each variable);

$$\text{If } \boldsymbol{x} \leq \boldsymbol{y} \Rightarrow F^{(n)}(\boldsymbol{x}) \leq F^{(n)}(\boldsymbol{y}) \tag{1}$$

ii. fulfills the boundary conditions;

$$\inf_{x\in\mathbb{I}^n} F^{(n)}(\boldsymbol{x}) = \inf \mathbb{I} \quad \text{and} \quad \sup_{x\in\mathbb{I}^n} F^{(n)}(\boldsymbol{x}) = \sup \mathbb{I} \tag{2}$$

$$\text{If } \mathbb{I} = [a, b],\ F^{(n)}(\boldsymbol{a}) = a \quad \text{and} \quad F^{(n)}(\boldsymbol{b}) = b; \\ \text{where } \boldsymbol{a} = (a, \ldots, a) \quad \text{and} \quad \boldsymbol{b} = (b, \ldots, b) \tag{3}$$

iii. for all $x \in \mathbb{I}$

$$F^{(1)}(x) = x. \tag{4}$$

2.1. OWA operator basics

An OWA operator (Yager, 1988) is a function $F_w: \mathbb{R}^n \to \mathbb{R}$, such that

$$F_w(\boldsymbol{a}) = \sum_{j=1}^{n} w_j a_{\sigma(j)} \tag{5}$$

where:

$\boldsymbol{a} \in \mathbb{R}^n$, S_n is the permutation group, $\sigma \in S_n$ is referred to as an ordering permutation, $a_{\sigma(i)} \geq a_{\sigma(i+1)}$ and the weight vector $w \in [0, 1]^n$ is normalized such that:

$$||w||_1 = \sum_{j=1}^{n} w_j = 1 \tag{6}$$

It can be easily shown that $F_{e_1} \geq F_w \geq F_{e_n}$, where e_i are the canonical basis vectors of $\mathbb{R}^n$, i.e.,

$$F_{e_1}(\boldsymbol{a}) = \max_{1\leq j\leq n} a_j,\ F_{e_n}(\boldsymbol{a}) = \min_{1\leq j\leq n} a_j \tag{7}$$

represent the logical 'or' and 'and' operators, respectively. Hereon, this property is referred to as max–min boundedness. The OWA operators are always commutative (neutral, symmetrical, anonymous),

$$F_w(\boldsymbol{a}) = F_w\left(a_{\tau(1)}, a_{\tau(2)}, \ldots, a_{\tau(n)}\right),\ \forall \tau \in S_n \tag{8}$$

monotonic (nonnegative responsive),

$$F_w(\boldsymbol{a}) \leq F_w(\boldsymbol{b}) \tag{9}$$

where:

$$a_i \leq b_i,\ i = 1, 2, \ldots, n$$

and idempotent (agreeing, unanimous, reflexive), $F_w(c, c, \ldots, c) = c,\ \forall c \in \mathbb{R}$. In fact, OWA operators have been characterized by the operators which satisfy the properties of commutativity, monotonicity, stability for the same positive linear transformations and ordered linkage. The definition of the last two properties, another characterization, and the corresponding proofs can be found in (Fodor, Marichal, & Roubens, 1995), being omitted here for brevity.

An OWA operator is neat if its value does not depend on the ordering of the aggregates, i.e.,

$$F_w(\boldsymbol{a}) = \sum_{j=1}^{n} w_j a_{\tau(j)},\ \forall \tau \in S_n \tag{10}$$

For example, the arithmetic mean ($w_i = 1/n$) is a neat OWA operator,

$$F_{AM}(\boldsymbol{a}) = \sum_{j=1}^{n} \left(\frac{a_j}{n}\right) \tag{11}$$

Two measures associated to OWA operators were also introduced by Yager (1988), both the dispersion (or entropy) and the maxness (or logical or-ness) of an OWA operator defined as

$$\text{Disp}(F_w) = -\sum_{j=1}^{n} w_j \ln\left(w_j\right) \tag{12}$$

where:

$$0 \leq \text{Disp}(F_w) \leq \ln n$$

and

$$\text{Maxness}(F_w) = \sum_{j=1}^{n} \frac{n-j}{n-1} w_j \tag{13}$$

where:

$$0 \leq \text{Maxness}(F_w) \leq 1,$$