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Decision Support

Competitive analysis of the online financial lease problem

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ABSTRACT

The financial lease is an important financing tool by which the lessee can acquire ownership of equipment upon the expiration of the lease after making a series of rent payments for the use of the equipment. In this paper, we consider an online version of this financial lease decision problem in which the decision maker (the lessee) does not know how long he/she will use the equipment. By assuming, the lessee can use the equipment through two options: financial lease or lease; we define and solve this online financial lease decision problem using the competitive analysis method. The optimal online strategies are discussed in each financial lease case with or without down payment. Finally, the optimal strategies are summarized as simple decision rules.

1. Introduction

The leasing industry has significant impacts on other industries, and has catalysed economic development in different countries (Peck, 2014; Vance, 2003). To decide if leasing is a profitable financing option, the decision maker should first determine the length of time the equipment will be used. However, in practice, the exact length of time for use of some equipment is usually unavailable, so researchers explore well-established techniques in the fully distribution-free model, i.e., the online model and competitive analysis (Albers, 2003; Borodin & El-Yaniv, 1998), to evaluate their strategies. This technique compares the results obtained with an online strategy to the result that could have been obtained if one had known the exact length of time in advance, with the latter scenario represented by an optimal offline strategy. Thus, the competitive analysis aims is to design an online strategy with the best possible worst-case guarantees. Competitive analysis has been used to study many problems within the fields of finance (El-Yaniv, 1998), operations research (Chen & Wang, 2015; Liu, Chu, Xu, & Zheng, 2010; Zheng, Cheng, Xu, & Liu, 2013) and management science (Larsen & Wøhlk, 2010).

Karp first formulated the classic "leasing problem" (a.k.a. the Ski-rental problem) using the following online model and competitive analysis method (Karp, 1992): assume the decision maker has two options, to lease or purchase the equipment. In the purchase option, a one-time cost is incurred, and thereafter usage is free of charge; in the lease option, the cost is proportional to usage time, and there is no one-time cost. The solution is straightforward with competitive ratio 2, i.e., the strategy never pays more than twice the optimum. There have been many generalizations of this simple problem. For example, El-Yaniv, Kaniel, and Linial (1999) incorporated interest rates into the model and determined the optimal online strategies. Irani and Ramanathan (1998) examined a situation in which the price of the equipment fluctuates while the rental cost stays constant. More recently, Lotker, Patt-Shamir, and Rawitz (2008a, 2008b) and Zhang, Ponn, and Xu (2011) proposed the multi-slope ski-rental problem and the multiple discount option ski-rental problem, respectively. Other classic variations of the problem include the replacement problem (El-Yaniv & Karp, 1997), the capital investment problem (Azar et al., 1999) and the Bahncard problem (Fleischer, 2001).

The lease option in *all* of the above research is an operating lease. Operating leases are contracts for rent. At the end of the contract period, the ownership of the rented equipment remains with the rental company. In practice, there is another important leasing category: the financial lease. Typically, in a financial lease, the lessee will find required equipment, and then contact a leasing company (the lessor) to arrange financing. Legal ownership of the equipment remains with the lessor until lease ends, at which time the ownership is transferred to the lessee (Vance, 2003).

In this paper, the decision maker has two options, a lease or financial lease, to acquire required equipment. In the former option, the decision maker pays a rental fee for the equipment, whose ownership

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remains with the rental company at any time. In the latter option, the decision maker either makes or does not make a down payment, and then pays financial lease fees during the length of time of the lease for the equipment, whose ownership will be transferred to the lessee upon the expiration of the lease. The problem then is how to decide the time to switch to a financial lease when the decision maker does not know how long he/she will use the equipment. We will call this problem *the online financial lease problem*. In this paper, the optimal strategies to solve the online financial lease problem in cases with and without a down payment will be presented, and the optimal strategies will be summarized as simple decision rules.

The following example illustrates a typical result based on the analysis presented in this paper.

Example

Consider someone wants to use a repertory. There are two options: lease with a rental fee at \$2000 per month, or a financial lease with a fee of \$3000 per month plus a down payment of \$10000 and in 24 months, the ownership will transfer to the lessee. Which policy guarantees the best performance for any possible length of time it is used in terms of a competitive analysis?

Answer

The following lease strategy guarantees a cost at most of 141.5 percent of the minimal possible cost, for any possible length of time: lease this repertory from the first to the 17th month and use the financial lease thereafter. On the other hand, if the lessee could negotiate with the rental company to decrease the down payment from \$10000 to zero, then the strategy to lease this repertory for one year and then use the financial lease the next year can guarantee a maximum cost of 133.3 percent of the minimum possible cost. In addition, the above two percentages are the minimum for each case, i.e., the two strategies are optimal. The details are provided in Sections 5 and 6. Note that there are no assumptions about the distribution of the length of time in use.

The rest of the paper is organized as follows. Section 2 provides the precise definition of the online financial lease problem, the competitive strategy, and the competitive ratio. Section 3 then presents the competitive strategy and the matching lower bound proof for the problem of a financial lease without down payment. Thus, the competitive strategy obtained in this section is optimal. Next, Section 4 describes the optimal competitive strategies for the problem of a financial lease with down payment. We discuss this problem with different cases, and for each we obtain the optimal strategy and optimal competitive ratio. Subsequently, Section 5 summarizes all results and provides simple decision rules for different cases of the online financial leasing problem. The following Section 6 provides numerical analysis results to illustrate the proposed models and strategies. Finally, Section 7 presents the concluding remarks and areas for future research.

2. Problem definition and notations

In this section, we present the precise definition of and notations for the online financial lease problem.

Formally, in our problem, the decision maker (the lessee) can use the equipment by lease or financial lease. That is to say, the lessee can lease the equipment at a cost of c per unit time, or can choose to use this equipment with a financial lease and pay $b \geq 0$ as a down payment, plus r for each unit time.

The following basic assumptions are used in the online financial lease problem,

- In financial lease, the lessee will obtain ownership of the equipment after z payments, and then no longer needs to pay the financial lease fees;
- The lessee can end a financial lease at any time, even just before obtaining ownership of the equipment. When the lessee decides to stop using the equipment within the lease period, the lease is terminated and the lessor is still the legal owner of the equipment, and the lessee does not need to pay the remaining fees;
- The length of time t for the use of the equipment is unknown.

Let $A(T)$ define the strategy wherein the decision maker leases the equipment from the start to time T, and uses a financial lease thereafter, so $A(0)$ represents the strategy that uses a financial lease from the start, and $A(\infty)$ indicates a perpetual lease. Note that this notation defines all possible strategies.

For an (unknown) length of time to use the equipment $t \geq 0$, define $Cost_{A(T)}(t)$ as all costs paid by strategy $A(T)$ from the start to t, including, if any, the cost of the financial lease fees at time t. We define $Cost_{opt}(t)$ as the minimum cost needed to cover the period from the beginning to t. A strategy $A(T)$ is said to be α-competitive if there exists two constants α, β such that

$$Cost_{A(T)}(t) \leq \alpha Cost_{opt}(t) + \beta \tag{1}$$

for all possible t, and this strategy is called an *online strategy* or a *competitive strategy*. Thus, our task is to acquire the equipment with minimal α in (1) for any unknown used time length t. All results in this paper hold under the stricter form of competitive analysis in which $\beta = 0$.

Since strategy $A(T)$ is only determined by parameter T, then our problem returns to how to determine the decision variable T. All notations introduced above are summarized in Table 1.

3. Financial lease without down payment

In this section, we examine the online financial lease problem without down payment, i.e., down payment $b = 0$. The analysis here illustrates the basic ideas for the subsequent general cases. Without loss generality, we consider $r > c$, otherwise the lessee should always choose to use the equipment via financial lease. Now, the cost of the optimal offline strategy is

$$Cost_{opt}(t) = \begin{cases} ct, & t < rz/c \\ rz, & t \geq rz/c \end{cases}$$

Before providing the optimal competitive strategy result, we present Lemmas 1 and 2 as follows.

Lemma 1. *Let $T^* = (r/c - 1)z$, the competitive ratio of online strategy $A(T^*)$ is $2 - c/r$.*

Proof. Recall that $A(T^*)$ represents the strategy using a lease during time T^* and then switching to a financial lease from then on. Thus, the cost of strategy $A(T^*)$ is

$$Cost_{A(T^*)}(t) = \begin{cases} ct, & t < T^* \\ cT^* + r(t - T^*), & T^* \leq t < T^* + z \\ cT^* + rz, & t \geq T^* + z \end{cases}$$

Then the ratio of costs between the online strategy $A(T^*)$ and optimal offline strategy is

$$\frac{Cost_{A(T^*)}(t)}{Cost_{opt}(t)} = \begin{cases} 1, & t < T^* \\ \frac{1}{t}(1 - \frac{r}{c})T^* + \frac{r}{c}, & T^* \leq t < z + T^*. \\ 2 - \frac{c}{r}, & t \geq z + T^* \end{cases}$$

The above ratio function is a piecewise continuous function, since $r > c$, the second function is an increasing function of t; thus, when $t = z + T^*$, that is $t = rz/c$, the above ratio approaches its greatest value