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Innovative Applications of O.R.

Near optimal design of wavelength routed optical networks

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ABSTRACT

The problem of designing a wavelength routed optical transport network without wavelength conversion at intermediate nodes is considered. A class of valid inequalities for wavelength routing and assignment is reported and is used to augment traditional network design formulations. The resulting network cost provides a lower bound on the cost of a network that permits wavelength routing. The resulting network is shown to be optimal for a majority of the problem instances tested and in those cases where it is not, a trial-and-error method is proposed that is able to find near-optimal solutions within relatively short period of time. This is achieved by developing efficient and effective heuristics that attempt to provide a feasible wavelength routing. Computational tests are reported on relatively larger problem sizes than have been reported in literature on the wavelength routing problem.

1. Introduction

We address the problem of designing a wavelength routed optical transport network. We are given a graph $G = (V, E)$, called the supply graph, and the demands units d_{ij} (integer valued) to be transported between each pair of nodes $i, j \in V$. An optical transport facility installed on edge $e \in E$ at cost c_e can carry C simultaneous optical signals via wavelength division multiplexing (WDM) technology using wavelengths λ_1 to λ_C. It is assumed that each unit of demand requires one wavelength. A demand may be routed from its origin to destination via several edges of E using optical switching at intermediate nodes. However, we assume that wavelength conversion at intermediate nodes is not permitted, and each demand must be assigned a specific wavelength λ_l, $1 \le l \le C$, which will be used on each edge of the end-to-end path of the demand. This is known in the literature as the *wavelength continuity constraint*. Another routing constraint is the non-splittability of demand units, i.e., each unit of demand must be carried along a single path, and cannot be split into smaller fractional units of size less than one unit and routed along several paths. However, if $d_{ij} > 1$, then it can be split into individual units, each unit being of size one. The objective is to design a minimum cost network that will permit a feasible wavelength assignment and routing for each demand unit. We call this problem WRND, or wavelength routed network design.

This problem is quite difficult to solve (the problem is NP-Hard since a special case of the problem, that of deciding whether a feasible wavelength routing with wavelength continuity exists on a given network, is NP-Complete (Refer Chlamtac, Ganz, & Karmi, 1992), but a relaxed version of this problem, called the standard network design problem (NDP), also known as the network loading problem, is relatively easier to solve although this problem is also NP-Hard (Bienstock, Chopra, Günlük, & Tsai, 1998). If the wavelength continuity constraint and non-splittability constraint are relaxed, WRND reduces to the NDP. In the NDP, we require a minimum cost installation of facilities, each with capacity C, on the edges of G, that will simply permit a feasible multicommodity flow of all demands. Although this problem is also quite hard to solve, some recent results have permitted optimal solutions of this problem for instances with 25–30 nodes when the facility cost function is Euclidean in nature (Agarwal, 2015).

Given an instance of the WRND, the optimal solution of the corresponding instance of the NDP is clearly a lower bound on the optimal solution of WRND. Our design approach for WRND makes use of this observation to find a near-optimal solution of WRND as follows. We solve the NDP relaxation of WRND, obtaining the optimal solution x with cost $Z(x)$. Given solution x, we attempt to find a feasible routing of demands in this network with non-splittability and wavelength continuity constraints imposed. If such a routing is indeed possible, then clearly, we have found the optimal solution of WRND. However, if a feasible routing is not achieved, let $n_\lambda > C$ denote the number of layers needed. We solve the problem NDP again with a reduced facility capacity $C' = C - \delta$, obtaining a solution x'_δ with cost $Z(x'_\delta)$. Clearly $Z(x'_\delta) \ge Z(x)$. By conducting a process of trial and error on

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the value of δ, we attempt to find a minimum cost network that is feasible for WRND. Once such a solution has been found, $Z(x'_{\delta}) - Z(x)$ represents the optimality gap of this solution. Using this approach, we were able to obtain solutions that were within 1.2% of the lower bound in most cases on a set of test problems with 25 nodes, 50 edges and traffic matrices of varying densities. Our computational tests (Section 6) suggest that for high traffic density problems, a good choice of $\delta = n_{\lambda} - C$, while for medium and low traffic density problems, a good choice of $\delta = n_{\lambda} - C + 1$ or $\delta = n_{\lambda} - C + 2$.

We note that the problem of finding a wavelength routing of given set of demands on a given network so as to minimize the number of wavelengths required (called routing and wavelength assignment or RWA) has received a great deal of research attention (Chen & Banerjee, 1995; Dutta & Rouskas, 2000; Ramaswami & Sivarajan, 1995; Zang, Jue, & Mukherjee, 2000). However, the problem of designing a minimum cost network that will permit wavelength routing of a given set of demands is largely unaddressed in the literature. Our main contribution is in addressing this gap.

Given the NDP solution x, the problem of finding a feasible wavelength routing (RWA) can be formulated in multiple ways depending on the specific problem context. Some of these formulations are reviewed in Section 2. Here we briefly outline the formulation used in this paper. This formulation is similar to the formulation by Lee, Kang, Lee, and Park (2002). Given a network, for each specific wavelength λ_l (called a λ-layer) we can route a subset of demands in this layer. The paths used by these demands must be non-overlapping because only one demand can use a given wavelength on each facility. This packing of demands into a specific λ-layer is represented by a column a_j, and the associated integer variable x_j, where j denotes the packing pattern, denotes the number of layers in the solution with this specific pattern of packing of demands. The formulation ensures that all demands are packed into one of the λ-layers, and the objective is to minimize the total number of layers needed to pack all demands. As there can be a very large number of ways to pack a layer, the formulation may have an extremely large number of columns. However, the LP relaxation of the problem can be solved using a column generation approach. Having solved the LP relaxation, we use a simple rounding heuristic to obtain an integer solution. If the total number of layers needed in this solution does not exceed C, we have found a feasible wavelength routing of all demands. Otherwise, information about the final number of layers needed is used to redesign the network with a suitable value of δ. We note that the column-generation subproblem is itself an integer program akin to a set-packing problem, and has a very large number of columns, each representing a demand path. This problem is also solved using a column generation approach and a rounding heuristic. However, the column generation subproblem in this case is the shortest path problem which is solved quite efficiently using Dijkstra's algorithm.

Although the RWA approach described above produces good results, it is computationally rather inefficient due to a 2-tier column generation structure. We propose several strategies to improve its computational efficiency by decoupling the wavelength assignment from the routing of demands. Computational results demonstrate that these strategies are extremely effective, and lead to many fold reduction in the computational effort without a significant deterioration in the solution quality. The details are given in Section 3.

The basic approach suggested for solving the WRND problem worked quite well in some cases, but in other cases the optimality gap turned out to be unacceptably large. A closer examination of these cases revealed that occasionally, the optimal topology designed for the NDP contains certain features which pose serious bottlenecks with respect to the wavelength continuity constraints. We augment the NDP formulation with additional binary variables and constraints so that such bottlenecks are prevented in the optimal solution. The constraints added are indeed valid for the WRND, and therefore, the resulting solution represents a valid lower bound. When used with this strengthened formulation, our approach seems to work extremely well. As a result, we are able to report solutions that are within 1.2% of optimality for a majority of problem instances as large as 25 nodes, 50 edges, fully-dense traffic matrices, and 160 λ-layers.

The rest of the paper is organized as follows. In Section 2, we place the problem in the context of extant literature. The wavelength routing model is described in Section 3. The strengthened topology design model is described in Section 4. Implementation details are provided in Section 5 and computational experience is detailed in Section 6. Some concluding remarks and scope for future research is given in Section 7.

2. Literature review

Wavelength division multiplexing is a well-explored area within telecommunication engineering. Good surveys on different aspects of the technology can be found in Sivalingam and Subramaniam (2000). The two separate subproblems of the WRND, i.e., NDP and RWA, have been independently well-studied. Recent advances in the NDP, using metric inequalities and p-partition based inequalities, (refer Agarwal, 2006; Avella, Mattia, & Sassano, 2007; Agarwal, 2015) coupled with new implementations of capacity formulation (Agarwal, 2013) are able to obtain optimal solutions to problems with up to 25–30 nodes in a few minutes of computing time, when the facility cost structure is Euclidean in nature.

Zang et al. (2000) provide a survey of different types of RWA problems that arise in WDM networks. In these problems, the underlying physical network is usually considered as given. Given this fixed physical network, the problem is to decide on wavelength routing and assignments to cater to a variety of objectives and constraints. The RWA problem considered in this paper is equivalent to the "static routing and wavelength assignment" problem discussed by Zang et al. (2000). Dutta and Rouskas (2000) provides a survey of formulations for the same problem with different objectives. Various heuristics are discussed. A key point of difference between such formulations and the formulation used in the present work is that in the former, the size of the formulation increases linearly with the number of wavelengths supported per fiber. In our model, as will be seen in Section 3, the size of the model is independent of the number of wavelengths supported per fiber.

Chen and Banerjee (1995) studied the RWA problem under the assumption of dynamic demands and limited number of wavelength converters in the network. Their approach relies on replicating the physical networks as many times as there are supported wavelengths. However, this can result in very large problem instances for higher number of wavelengths per fiber. Ramaswami and Sivarajan (1995) studied the RWA problem with the objective of maximizing the amount of carried traffic subject to wavelength continuity constraints. Linear programming-based bounds are developed. Ramaswami and Sivarajan (1996) considered the RWA problem with demands specified as traffic patterns (unlike specification in absolute units as considered in the present work) and constraints on propagation delays along paths. Propagation delays are not considered in our work, but can be easily incorporated, if required. Numerical results are presented based on the analysis of 6-node network topologies. Mukherjee, Banerjee, Ramamurthy, and Mukherjee (1996) studied the RWA problem in the context of two objectives — minimizing delay or maximizing the offered load. Sasaki (2000) discussed the RWA problem in cases of networks with specific underlying physical topology such as rings, trees and line networks. The author discusses various upper bounds on the number of wavelengths needed per fiber to support different traffic types. Chlamtac et al. (1992) studied the computational complexity of assigning wavelengths to lightpaths, each corresponding to a predefined set of contiguous arcs in the physical network, and showed that this problem is NP-Hard. Banerjee and Mukherjee (1997) developed a model that minimizes the average hop distance ignoring the wavelength-continuity constraint.