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A branch-and-cut algorithm for the profitable windy rural postman problem

Thais Ávila [a], Ángel Corberán [a], Isaac Plana [b], José M. Sanchis [c,*]

[a] Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Spain
[b] Departamento de Matemáticas para la Economía y la Empresa, Universidad de Valencia, Spain
[c] Departamento de Matemática Aplicada, Universidad Politécnica de Valencia, Spain

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ABSTRACT

In this paper we study the profitable windy rural postman problem. This is an arc routing problem with profits defined on a windy graph in which there is a profit associated with some of the edges of the graph, consisting of finding a route maximizing the difference between the total profit collected and the total cost. This problem generalizes the rural postman problem and other well-known arc routing problems and has real-life applications, mainly in snow removal operations. We propose here a formulation for the problem and study its associated polyhedron. Several families of facet-inducing inequalities are described and used in the design of a branch-and-cut procedure. The algorithm has been tested on a large set of benchmark instances and compared with other existing algorithms. The results obtained show that the branch-and-cut algorithm is able to solve large-sized instances optimally in reasonable computing times.

1. Introduction

Routing problems with profits have been widely studied in recent years. They deal with situations where the customers to be served are not known a priori but have to be selected from a set of potential customers that have an associated profit that is collected if the customer is served. Basically, they consist of finding one or more routes that (jointly) serve the selected customers in such a way that an objective function of the profit and/or the cost is optimized.

If the customers are represented as nodes in a graph we have the so called node (or vehicle) routing problems with profits. In Feillet, Dejax, and Gendreau (2005a) these problems are classified according to their objective functions. A problem is called *profitable* when it consists of finding a tour that maximizes the difference between the total collected profit and the traveling cost. In the *orienteering* problem, the objective is to maximize the collected profit with the constraint that the cost of the tour does not exceed a given limit. Finally, in the *prize-collecting* problem, we look for a minimum cost tour collecting at least a given amount of profit. A good number of papers are available on node routing problems with profits. Among them, we refer the reader to the surveys Feillet et al. (2005a) and Vansteenwegen, Souffriau, and Oudheusden (2011), and the more recent one Archetti, Speranza, and Vigo (2014, chapter 10). In this last paper, several classes of applications that can be modeled by means of a routing problem with profits are presented.

Arc routing problems with profits refer to those problems where the customers are represented as arcs and/or edges of a graph. Similarly to node routing problems, they can be classified as profitable, prize-collecting or orienteering arc routing problems (see Archetti and Speranza, 2014, chapter 12). Although the number of published papers on arc routing problems with profits is smaller than for their node routing counterparts, this is a growing area that has recently attracted a good number of researchers that have studied many of its variants.

As far as we know, Malandraki and Daskin (1993) were the first authors to study such a problem. They introduced the Maximum Benefit Chinese Postman Problem on a directed graph. In this problem, several profits are associated with each arc, one for each time the arc is traversed with a service, and the objective is to find a tour with maximum profit. In Pearn and Chiu (2005) and Pearn and Wang (2003), some heuristic algorithms for its solution on directed and undirected graphs are proposed, while in Corberán, Plana, Rodríguez-Chía, and Sanchis (2013), the polyhedron associated with the problem defined on an undirected graph is studied and a branch-and-cut algorithm is proposed. Archetti and Speranza (2014, chapter 12) suggest a new name for the Maximum Benefit Chinese Postman Problem more according to the recent classification, the Profitable Rural Postman Problem with Multiple Visits (PRPPMV). We also think it is

* Corresponding author. Tel.: +34 963877000.
E-mail addresses: thais.avila@uv.es (T. Ávila), angel.corberan@uv.es (Á. Corberán), isaac.plana@uv.es (I. Plana), jmsanchis@mat.upv.es (J.M. Sanchis).

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a more appropriate name. In the Prize-Collecting Arc Routing Problem (also called Privatized Rural Postman Problem) only the edges in a given subset have an associated profit, which can be collected just once. This problem was introduced in Aráoz, Fernández, and Zoltan (2006) and an exact algorithm for solving it on undirected graphs is described in Aráoz, Fernández, and Meza (2009). Note that this problem, which should be called the (Undirected) Profitable Rural Postman Problem (PRPP), is a special case of the PRPPMV when some of the edges have a positive profit while the remaining ones have zero profit. In Schaeffer, Ríos-Mercado, and Fernández (2014), Schaeffer et al. describe an ant-colony heuristic for the same problem defined on a windy graph. Authors call this problem the Windy Prize-Collecting Rural Postman Problem, but, according to Archetti and Speranza (2014, chapter 12), it should be called the Profitable Windy Rural Postman Problem, which is the name we have used in this article.

Other arc routing problems with profits with a single vehicle are those studied in Aráoz, Fernández, and Franquesa (2009), Guastaroba, Mansini, and Speranza (2009), Corberán, Fernández, Franquesa, and Sanchis (2011), Vansteenwegen et al. (2011), Aráoz, Fernández, and Franquesa (2013), Black, Eglese, and Wøhlk (2013), Archetti, Guastaroba, and Speranza (2014), Benavent, Corberán, Gouveia, ao, and Pinto (2015), Colombi and Mansini (2014), and Archetti, Corberán, Plana, Sanchis, and Speranza (2015). If multiple vehicles are available, we find the papers Feillet, Dejax, and Gendreau (2005b), Archetti, Feillet, Hertz, and Speranza (2010), Euchi and Chabchoub (2011), Zachariadis and Kiranoudis (2011), Cura (2013), Archetti, Corberán, Plana, Sanchis, and Speranza (2014), and Archetti, Corberán, Plana, Sanchis, and Speranza (2013). The reader is referred to the survey Archetti and Speranza (2014, chapter 12) for more details on the problems described in the above papers.

Here we study the Profitable Windy Rural Postman Problem (PWRPP). This problem is a generalization of the well known Rural Postman Problem (RPP), which consists of finding a minimum cost tour in an (undirected) graph traversing at least once all the edges in a subset of required edges. If we consider a windy graph, in which each edge has two costs corresponding to the two directions of traversal, the problem is known as the Windy RPP. If, in addition to the required edges, there is a subset of (required) vertices to be visited, the problem is known as the Windy General Routing Problem (WGRP). Both problems have been extensively studied (see Corberán, Plana, & Sanchis, 2007; 2008; Plana, 2005). Moreover, some real-life applications have been modeled as pure WRPPs or variants. We refer the reader to the recently published book on arc routing problems (Corberán & Laporte, 2014) for more details on this. We just want to mention the recent papers by Dussault, Golden, Groër, and Wasil (2013) and Dussault, Golden, and Wasil (2014) on practical extensions of the WRPP in the context of snow plowing. The first work is motivated by the fact that deadhead travel over streets that have already been plowed is significantly faster than the time it takes to plow the street. Moreover, on some steep streets, it is much more difficult, or impossible, to plow uphill. This problem, called the Plowing with Precedence Problem, differs from most arc routing problems because the cost of traversing a street changes depending on the order of the streets on a route. The second paper deals with the problem of determining a set of routes for snow plows that minimize the maximum route length.

While in the WRPP there is a fixed subset of edges that have to be traversed by the vehicle, in the PWRPP the route does not have to traverse any specific subset of edges. Instead, there is a profit associated with some edges that is collected the first time the edge is traversed. The objective is to find the route that maximizes the sum of the profit collected minus the traversal costs. In this article, we first give a precise definition of the problem and model it as an integer program. In Section 3, we study the polyhedron associated with the PWRPP and propose several families of facet-defining inequalities. Section 4 describes the branch-and-cut algorithm we have designed and implemented for the PWRPP solution, while Section 5 presents the computational results obtained on a large set of benchmark instances. Finally, some conclusions are presented in Section 6.

2. Problem definition and formulation

The profitable windy rural postman problem can be defined as follows. Let $G = (V, E)$ be an undirected connected graph, where vertex 1 represents the depot and there are two non-negative costs, (c_{ij}, c_{ji}), associated with each edge $e = (i, j) \in E$, corresponding to the costs of traversing e from i to j and from j to i, respectively. Moreover, associated with each edge e in a subset $E_B \subseteq E$, there is a profit $b_e > 0$, which is collected the first time e is traversed (serviced). The objective is to find a closed walk on G, starting and ending at the depot, that maximizes the net profit (the total collected profit minus the total traversal cost).

We will use the following notation. Given two vertex sets $S, T \subseteq V$ we define:

$$(S : T) = \{e = (i, j) \in E : \ i \in S, \ j \in T \text{ or } i \in T, \ j \in S\},$$
$$\delta(S) = (S : V \setminus S),$$
$$E(S) = \{(i, j) \in E : i, j \in S\}.$$

The above sets can be referred only to the edges in E_B as $(S{:}\ T)_B$, $E_B(S)$ and $\delta_B(S)$.

In order to formulate the problem, we introduce the following variables:

- For each edge $e = (i, j) \in E$, x_{ij} and x_{ji} denote the number of times e is traversed from i to j and from j to i, respectively,
- For each edge $e = (i, j) \in E_B$, y_e is a binary variable that takes value 1 if e is serviced and 0 otherwise.

Hence, to each closed walk for the PWRPP we associate an incidence vector $(x, y) \in \mathbb{Z}^{2|E|+|E_B|}$. In what follows we use the word "tour" to refer to both the closed walk and to its incidence vector. Given $F \subseteq E$, we define

$$x(F) = \sum_{(i,j)\in F} (x_{ij} + x_{ji}),$$

and given $S, T \subseteq V$,

$$x(S, T) = \sum_{i\in S, j\in T} x_{ij}.$$

In particular, $x(\delta^+(S)) = x(S, V \setminus S)$ and $x(\delta^-(S)) = x(V \setminus S, S)$ denote the sum of the variables in x 'leaving' and 'entering' in S, respectively.

Then, the PWRPP can be formulated as follows:

$$\text{Maximize} \quad \sum_{e\in E_B} b_e y_e - \sum_{(i,j)\in E} (c_{ij}x_{ij} + c_{ji}x_{ji})$$

s.t.:

$$x(\delta^+(i)) = x(\delta^-(i)), \quad \forall i \in V \tag{1}$$

$$x_{ij} + x_{ji} \ge y_e \quad \forall e = (i, j) \in E_B \tag{2}$$

$$x(\delta^+(S)) \ge y_e, \quad \forall S \subset V\setminus\{1\}, \quad \forall e \in E_B(S)\cup\delta_B(S) \tag{3}$$

$$x_{ij}, \quad x_{ji} \ge 0, \quad \forall (i, j) \in E \tag{4}$$

$$0 \le y_e \le 1 \quad \forall e \in E_B \tag{5}$$

$$x_{ij}, x_{ji}, y_e \text{integer}, \quad \forall e = (i, j) \in E \tag{6}$$

Eq. (1) are the well known symmetry conditions. Traversing inequalities (2) guarantee that each serviced edge is traversed, while