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# The single-item green lot-sizing problem with fixed carbon emissions

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ABSTRACT

We consider in this paper a single-item lot sizing problem with a periodic carbon emission constraint. In each period, the carbon emission constraint defines an upper limit on the average emission per product. Different modes are available, each one is characterized by its own cost and carbon emission parameters. The problem consists in selecting the modes used in each period such that no carbon emission constraint is violated, and the cost of satisfying all the demands on a given time horizon is minimized. This problem has been introduced in Absi et al. (2013), and has been shown polynomially solvable when only unit carbon emissions are considered. In this paper, we extend the analysis for this constraint to the realistic case of a fixed carbon emission associated with each mode, in addition to its unit carbon emission. We establish that this generalization renders the problem NP-hard. Several dominant properties are presented, and two dynamic programming algorithms are proposed. We also establish that the problem can be solved in polynomial time for a fixed number of modes when carbon emission parameters are stationary.

## 1. Introduction

Considering sustainability issues in Supply Chain Management is becoming more and more important (Linton, Klassen, & Jayaraman, 2007). In particular, optimizing environmental objectives or managing environmental constraints are associated with the concept of Green Supply Chain (Srivastava, 2007). Recently, various researchers have studied how to consider carbon emissions in production and distribution planning problems. One of the seminal work in the domain can be found in Benjaafar, Li, and Daskin (2013), where the authors propose a mathematical model which includes a global carbon emission constraint on the planning horizon. They perform a numerical study to derive some managerial insights. Based on the same type of constraints, Helmrich, Jans, van den Heuvel, and Wagelmans (2015) show that the problem is NP-hard and propose various solution methods, including a Lagrangian heuristic and a Fully Polynomial Time Approximation Scheme (FPTAS). A global carbon emission constraint is also considered in Velázquez-Martínez, Fransoo, Blanco, and Mora-Vargas (2014), and models for different scenarios are discussed. Carbon emission constraints are taken into account in static inventory and distribution problems (such as in Arıkan & Jammernegg, 2014; Konur, 2014). Various authors have also studied how to take carbon emissions into account in the objective function, either in static inventory models (such as in Bouchery, Ghaffari, Jemai, & Dallery, 2012; Chen, Benjaafar, & Elomri, 2013; Konur & Schaefer, 2014; Toptal, Özlü, & Konur, 2014) or in dynamic lot-sizing models (such as in Palak, Ekşioğlu, & Geunes, 2014; Romeijn, Morales, & Van den Heuvel, 2014).

Absi et al. (2013) propose four types of carbon emission constraints in multi-mode dynamic lot sizing: (1) Periodic carbon emission constraint, (2) Cumulative carbon emission constraint, (3) Global carbon emission constraint and (4) Rolling carbon emission constraint. Compared to the carbon emission constraints considered in most papers, these constraints impose a maximum value not on the carbon emissions, but on the average carbon emission per product. This type of constraints is particularly relevant to the firms who want to display the carbon footprint of their products. Notice that these type of constraints do not limit the supply capacity since it is always possible to select a supply mode that can satisfy the constraints. Another advantage, except for the global carbon emission constraint,

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is that the constraints do not strongly depend on the length of the horizon (see Absi et al., 2013 for a more extensive discussion). The uncapacitated single-item problem with Periodic Carbon emission constraint, called ULS-PC in this paper, is shown polynomial, and a dynamic programming algorithm is proposed (Absi et al., 2013).

In this paper, we analyze how fixed carbon emissions impact the problem with periodic carbon emission constraints. A fixed carbon emission is incurred at each period a mode is selected, and corresponds for instance to the activities associated with packaging the products for the associated mode. A mode corresponds to the combination of a production facility and a transportation mode for supplying products. This problem is called ULS-FPC in the following. The problem consists in selecting in each period the modes to use and the quantities to order such that the supplying costs and the inventory costs are minimized, while satisfying in each period a carbon emission constraint per product.

The paper is organized as follows. The problem is formally introduced and modeled in Section 2. It is important structural properties are presented in Section 3. We show in Section 4 that the problem is NP-hard. The special case where carbon emission parameters are stationary is studied in Section 5, and two dynamic programming algorithms are proposed. Finally, the paper ends with some conclusions and perspectives in Section 6.

## 2. Problem modeling

We are interested in optimizing the supply (production and transportation) plan (when and how much to supply) of an item to satisfy a deterministic time-dependent demand over a planning horizon of $T$ periods. Let us consider $M$ different supplying modes associated with different available production locations and transportation modes. Costs to be minimized include holding cost and unitary and fixed supplying costs which depend on the supplying mode. We study the problem with periodic carbon emission constraints considering $ef_t^m$, which is the fixed environmental impact associated with mode $m$ in period $t$. This parameter is independent of the supplied quantity.

The parameters and variables of the multi-sourcing lot-sizing problem are formally defined below.

**Parameters:**

$d_t$: Demand in period $t, t = 1, \dots, T$,
$h_t$: Unitary holding cost at the end of period $t$,
$p_t^m$: Unitary supplying cost of mode $m$ in period $t$,
$f_t^m$: Supplying setup cost of mode $m$ in period $t$,
$ev_t^m$: Environmental impact (carbon emission) related to supplying one unit using mode $m$ in period $t$,
$ef_t^m$ Fixed environmental impact related to using mode $m$ in period $t$,
$E_t^{\max}$: Maximum unitary environmental impact allowed in period $t$.
$\bar{ev}_t^m = ev_t^m - E_t^{\max}$: Relative environmental impact for mode $m$ in period $t$ (negative for ecological modes, positive for non-ecological modes).

**Variables:**

$x_t^m$: Quantity supplied in period $t$ using mode $m$,
$y_t^m$: Binary variable which is equal to 1 if mode $m$ is used in period $t$, and 0 otherwise,
$s_t$: Inventory carried from period $t$ to period $t+1$.

The carbon emission constraint of the ULS-FPC problem is a generalization of the one defined by Absi et al. (2013) for the ULS-PC. It ensures that the average amount of carbon emission at any period $t$ in addition to the fixed consumption is lower than or equal to the maximum unitary carbon emission. Hence, the unused amount of carbon emission in a given period cannot be used in the following periods. More formally, this tight constraint can be defined as follows for each period $t$:

$$\frac{\sum_{m=1}^{M}\left(ev_t^m x_t^m + ef_t^m y_t^m\right)}{\sum_{m=1}^{M} x_t^m} \le E_t^{\max}$$

The mathematical formulation of the multi-sourcing lot-sizing problem, with fixed and periodic carbon emission constraints, is given below:

$$\min \quad \sum_{m=1}^{M}\sum_{t=1}^{T}\left(p_t^m x_t^m + f_t^m y_t^m\right) + \sum_{t=1}^{T} h_t s_t \qquad (1)$$

$$s.t. \quad \sum_{m=1}^{M} x_t^m - s_t + s_{t-1} = d_t, \quad t = 1, \dots, T \qquad (2)$$

$$x_t^m \le B y_t^m, \quad t = 1, \dots, T, m = 1, \dots, M \qquad (3)$$

$$\sum_{m=1}^{M}\left(\bar{ev}_t^m x_t^m + ef_t^m y_t^m\right) \le 0, \; t = 1, \dots, T \qquad (4)$$

$$x_t^m \in \mathbb{R}^+, y_t^m \in \{0, 1\}, \quad t = 1, \dots, T, m = 1, \dots, M$$

$$s_t \in \mathbb{R}^+, \quad t = 1, \dots, T$$

The objective function (1) minimizes the fixed and variable production and transportation costs and the total holding cost. Constraints (2) are the inventory balance equations, and Constraints (3), in which $B$ is a big value, ensure that an item cannot be supplied using mode $m$ at period $t$ if $m$ is not one of the selected modes. The parameter $B$ must be calculated according to the sum of the demands ($\sum_{t=1}^{T} d_t$) and threshold parameters that will be introduced later. The carbon emission constraints are defined by (4).

Due to Constraints (4), the existence of a feasible solution cannot be guaranteed.

**Property 1.** A feasible solution exists if and only if at least one of the following conditions holds for periods $t' \le t$ where $t$ is the first period with a strictly positive demand:

- $\bar{ev}_{t'}^m < 0$ for at least one $t'$
- $\bar{ev}_{t'}^m = 0$ and $ef_{t'}^m = 0$ for at least one $t'$

**Proof.** If one of the two conditions is verified, it is always possible to satisfy the total demand from period $t'$ by setting $x_{t'}^m \ge \sum_{k=t}^{T} d_k$ such that $\bar{ev}_{t'}^m x_{t'}^m + ef_{t'}^m \le 0$. Conversely, if $\bar{ev}_{t'}^m > 0$ for all $m$ and $t'$, $\bar{ev}_{t'}^m x_{t'}^m + ef_{t'}^m > 0$ whatever $m$, $t'$ and $x_{t'}^m > 0$. □

In the remainder of the paper, we consider that the feasibility of the problem is always guaranteed.

## 3. Structural properties of optimal solutions

The ULS-PC problem is a particular case of the ULS-FPC problem, with no fixed carbon emission (Absi et al., 2013). In this section, we recall properties that hold for both ULS-PC and ULS-FPC, and state new ones. These properties will be used in Section 5 to derive dynamic programming algorithms to solve the ULS-PC problem in the stationary case.

Recall that the periodic carbon emission constraint ensures that, in each period $t$, the average amount of carbon emission per product ordered does not exceed the impact limit $E_t^{\max}$. As we assume that the fixed carbon emission parameter is non-negative, clearly, at least one ecological mode $m$ must be chosen if a quantity is supplied in period $t$. In Absi et al. (2013), it was shown that solutions using at most two modes in each period are dominant for ULS-PC. This dominance property still holds when considering fixed carbon emissions.

**Theorem 1.** *There exists an optimal solution for the ULS-FPC problem that uses at most two modes in each period.*

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