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Most productive scale size versus demand fulfillment: A solution to the capacity dilemma

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ABSTRACT

The field of economics associates capacity planning with economic scale size and finds the characteristics of the production function whereas the operations management community focuses on demand fulfillment to reduce the loss of sales or inventory for profit maximization. However, there is a troublesome capacity trade-off for firms that need to achieve economic scale size and demand fulfillment simultaneously; in particular, a firm's demand is variable and some of the variation is random. This study proposes a multi-objective mathematical program with data envelopment analysis (DEA) constraints to set an efficient target which shows a trade-off between the most-productive-scale-size (MPSS) benchmark and a potential demand fulfillment benchmark. The study also employs the minimax regret (MMR) approach and the stochastic programming (SP) technique to address target variations caused by demand fluctuations. The result shows how capacity planning via the proposed models can help managers address the capacity dilemma.

1. Introduction

The capacity of a plant is defined as the maximum number of units that the plant can produce in a given time (Nahmias, 2009). The goal of capacity planning is to provide the supply that exactly matches the level of demand. However, due to the nature of demand fluctuation, capacity planning becomes a complex issue since capital investment is irreversible or costly (Abel & Eberly, 1996).

One important issue related to capacity planning decisions is: should we base our capacity planning on our firm's economic scale size or on chasing the demand? The field of economics says that determining the optimal scale, in particular, the most productive scale size (MPSS), presenting the point maximizing the ratio of total output to total input, implies a potential cost advantage, because the fixed costs are spread out over more units of output when increasing scale size and the average product goes down after the MPSS point (Banker, 1984). On the other hand, the operations management community favors demand fulfillment, which can reduce the mismatch, which causes either a capacity shortage or a capacity surplus, between supply and demand. Demand fulfillment leads to revenue maximization, because selling more products creates profits and conserves resources. However, in practice, demand fluctuation causes a gap between supply and demand; thus, when the firm may not achieve MPSS and demand fulfillment objectives simultaneously, operations management encounters a capacity dilemma (i.e., a trade-off between MPSS and demand fulfillment).

MPSS can be identified on the efficient surface of the production possibility set (PPS), which is defined by the production function. The production function is the function on the set of inputs whose value is the maximum possible output for a given set of inputs. It is nonnegative, nondecreasing, and a firm produces zero output when all inputs are zero (Coelli, Prasada Rao, O'Donnell, & Battese, 2005). Banker (1984) defined that the MPSS for a given input and output mix is the scale size at which the outputs produced per unit of the inputs is maximized. He showed that MPSS is equivalent to the benchmark on a constant-returns-to-scale (CRS) frontier, i.e., when increasing all inputs results in the same proportional increase in output. Let $\boldsymbol{x} \in \mathbb{R}_+^I$ denote the input vector and $\boldsymbol{y} \in \mathbb{R}_+^J$ denote the output vector of the production system. Define the production possibility set (PPS) as $T = \{(\boldsymbol{x}, \boldsymbol{y}) : \boldsymbol{x} \text{ can produce } \boldsymbol{y}\}$. Thus, based on Banker's definition, the point $(\boldsymbol{x}, \boldsymbol{y}) \in T$ is MPSS if and only if for every $(a\boldsymbol{x}, b\boldsymbol{y}) \in T$ we have $a \geq b$ (Banker, Cooper, Seiford, Thrall, & Zhu, 2004; Khodabakhshi, 2009). In other words, marginal product may be increasing for small values of input but must be diminishing for values of input exceeding those of any point in MPSS (Lee & Johnson, 2013).

In the previous studies, Banker (1984) described a data envelopment analysis (DEA) method identifying the MPSS via CRS efficient frontier. It implies that a DEA model gives a MPSS set containing a range of points based on different input mixes and output mixes. Banker, Chang, and Cooper (1996) developed a modified CRS DEA

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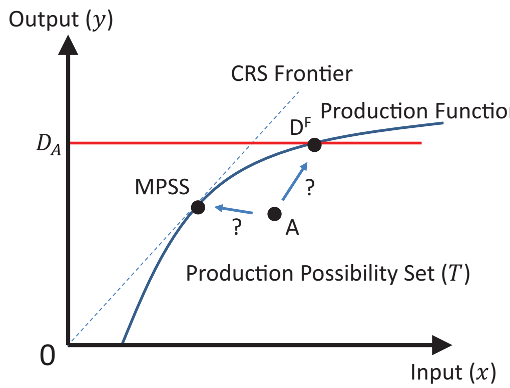

Fig. 1. Capacity dilemma.

model that can identify the MPSS, however, this method may provide multiple optimal solutions and the target cannot be determined uniquely. Appa and Yue (1999), who provided different DEA formulations that set a unique scale target under both input-oriented and output-oriented DEA models with single output, indicated that their unique scale targets corresponded to either the largest MPSS (i.e., a MPSS point with the maximal output) or the smallest MPSS (i.e., a MPSS point with the minimal output). An alternative approach by Zhu (2000) can control to identify the largest MPSS or the smallest MPSS. Fukuyama (2003) extended the scale return notions into a directional distance function framework (Chambers, Chung, & Färe, 1996) by generalizing Banker's MPSS concept and presenting a directional technology scale elasticity formula.

Fig. 1 shows a capacity dilemma between MPSS and demand fulfillment. The figure illustrates for one input and one output the simplest general case of the production function with a minimum scale of input below which output is not possible. Since the true production function cannot be observed, the variable-returns-to-scale (VRS) frontier is fitted and is good enough for practical purposes to approximate the frontier from limited data. Firm A faces a capacity dilemma on a single-input and single-output production function. The curve describes the production function and the dashed line is the CRS frontier that identifies the MPSS benchmark (Banker, 1984). D_A indicates Firm A's forecast demand and truncates the production function by the set of points $\boldsymbol{D}^F$ (i.e., demand fulfillment). Since Fig. 1 is an illustration example and we do not consider the time or seasonal effect, the forecast demand is just the estimated total demand over a planning period of interest. Note that different firms may have different forecast demands. Located below the production function, Firm A represents inefficiency and faces a capacity dilemma between MPSS and demand fulfillment. Firm A should attempt to adjust inputs and outputs to move to a point on the production frontier. But any point on the frontier between its MPSS benchmark and $\boldsymbol{D}^F$ is plausible.

Discussing "MPSS versus demand fulfillment" provides interesting managerial insights like "profitability versus profit". Profit is the difference between revenue and cost, and profitability is the ratio of revenue to cost. From the perspective of production economics, profitability is a more reasonable index to assess productivity, because the profitability function is homogenous of degree zero in prices. Namely, while the price doubles, the profit doubles, but the profitability does not change. This unscaled nature of profitability is similar to productivity and represents the input-to-output performance (Lee & Johnson, 2012). It implies that MPSS is insensitive to demand variation and provides the justification for a risk-averse decision-maker to set a target toward MPSS in order to reduce uncertainty, unlike a risk-seeking decision-maker who prefers chasing demand for profit maximization.

This study proposes a multi-objective decision analysis (MODA) model embedded with DEA constraints to release the dilemma and provide a compromise solution. We consider the demand factor and use mathematical programming to identify an efficient benchmark on the production function that represents a trade-off between MPSS and demand fulfillment. Due to the random demand and limited information about the distribution of possible values, we propose three approaches to address decision under uncertainty without the probability distribution of forecast demand: a minimax regret (MMR) (Savage, 1951) provides a conservative target to avoid the worst case via regret quantification, an expected value (EV) technique calculates the expected value of forecast demands based on the principle of indifference (Keynes, 1921) (i.e., all demand scenarios occur with equal probability), and a stochastic programming (SP) finds a robust solution based on the principle of indifference.

This study makes two contributions to the literature. First, we address the capacity dilemma between the MPSS and demand fulfillment by combining DEA and MODA to develop a compromise solution. We set a target on the production function and move toward it in order to guide capacity planning decisions. The trade-off choice between a MPSS (cost-oriented) strategy and a demand-chasing (revenue-oriented) strategy shows the risk preference of the decision-maker. Second, we develop the MMR model, EV model and SP technique when addressing decision under uncertainty in forecast demand and the solution comparison of the proposed models supplies useful managerial insights.

The remainder of this paper is organized as follows. Section 2 introduces a DEA model to identify MPSS. Section 3 proposes a compromise target between MPSS and deterministic demand and discusses three separate cases. Section 4 considers demand uncertainty and introduces the MMR model and SP technique. Section 5 gives a numerical example, and Section 6 concludes.

2. Most productive scale size

As mentioned, Banker (1984) shows that MPSS is equivalent to the efficient benchmark on a constant-returns-to-scale (CRS) frontier, as shown in Fig. 1. This study considers the single-output production function. The PPS can be estimated by DEA (Banker, Charnes, & Cooper, 1984). In particular, CRS DEA can be used to identify the MPSS and variable-returns-to-scale (VRS) can be used to estimate piecewise linear concave frontier enveloping all observations. We describe VRS DEA and CRS DEA as follows.

First, we estimate the variable-returns-to-scale (VRS) frontier. It is the frontier of the production possibility set whose elements produce no more output using at least as much input as some convex combination of observations. There are K firms that produce single output from I inputs. The ith input and single output for firm k are denoted X_{ik} and Y_k. λ_k is the convex-combination multiplier of the kth firm. The VRS PPS $\tilde{T}^{\text{VRS}}$ is defined by Eq. (1).

$$\tilde{T}^{\text{VRS}} = \left\{ (\boldsymbol{x}, y) \left| \begin{array}{c} \sum_{k=1}^{K} \lambda_k Y_k \geq y; \\ \sum_{k=1}^{K} \lambda_k X_{ik} \leq x_i, \forall i \in \{1, \ldots, I\}; \\ \sum_{k=1}^{K} \lambda_k = 1; \\ \lambda_k \geq 0, \forall k \in \{1, \ldots, K\} \end{array} \right. \right\} \tag{1}$$

Define the distance function $D^{\text{VRS}}(\boldsymbol{x}, y) = \min\{\theta | (\theta \boldsymbol{x}, y) \in \tilde{T}^{\text{VRS}}, \theta \leq 1\}$. Thus, for a specific firm k, we can measure the VRS input-oriented efficiency $\theta_k^{\text{VRS}} = D^{\text{VRS}}(\boldsymbol{x}_k, y_k)$, and a point $(\boldsymbol{x}_k, y_k)$ is on the VRS efficient frontier if $D^{\text{VRS}}(\boldsymbol{x}_k, y_k) = 1$, where $\boldsymbol{x}_k = (X_{1k}, \ldots, X_{Ik})$.