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The traveling salesman problem with time-dependent service times

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ABSTRACT

This paper introduces a version of the classical traveling salesman problem with time-dependent service times. In our setting, the duration required to provide service to any customer is not fixed but defined as a function of the time at which service starts at that location. The objective is to minimize the total route duration, which consists of the total travel time plus the total service time. The proposed model can handle several types of service time functions, e.g., linear and quadratic functions. We describe basic properties for certain classes of service time functions, followed by the computation of valid lower and upper bounds. We apply several classes of subtour elimination constraints and measure their effect on the performance of our model. Numerical results obtained by implementing different linear and quadratic service time functions on several test instances are presented.

1. Introduction

The purpose of this paper is to introduce, model and solve a version of the classical traveling salesman problem (TSP) with time-dependent service times (TSP-TS), an extension of the asymmetric TSP. Öncan, Altınel, and Laporte (2009), and Roberti and Toth (2012) present comprehensive reviews of the available mathematical formulations for the asymmetric TSP, some of which will be extended to model the TSP-TS.

In most of the research on the TSP, service times are either ignored, or assumed to be constant and thus accounted for in travel times. However in practice, it can easily be observed that service times vary according to several factors which naturally depend on the time of day (e.g., availability of parking spaces, accessibility to the customer at its location, and so on). In the TSP-TS, the service time required at each customer is not fixed a priori, but depends on the start time of service. The TSP-TS aims to minimize the total route duration including the total travel time and the total service time. This problem can formally be defined on a connected digraph $G = (N, A)$. In this graph, $N = \{0, 1, \ldots, n, n+1\}$ is the set of nodes and $A = \{(i, j) \mid i, j \in N, i \neq j\}$ is the set of arcs. Nodes 0 and $n+1$ correspond to the starting and ending points of the salesman's tour, respectively. Each node in $N \setminus \{0, n+1\}$ corresponds to a distinct customer. With each arc (i, j) in A is associated a travel time t_{ij}. Each customer i has a service time defined as a continuous function $s_i(b_i)$, where b_i corresponds to the start time of service at that customer location.

In the TSP literature, time-dependency is usually addressed in terms of travel times. The interested reader is referred to Gouveia and Voß (1995) who present a classification of formulations proposed for the time-dependent TSP. Picard and Queyranne (1978), Vander Wiel and Sahinidis (1996), and Bigras, Gamache, and Savard (2008) consider a time-dependent TSP in which the travel time between any two nodes depends on the time period of the day. It is further assumed that when the salesman starts traversing an arc, no transition from one time period to the next takes place during this travel, in other words there is no transient zone. More specifically, the travel time from node i to node j depends on the time period in which node i is visited. This problem with discrete travel times can be viewed as a single machine scheduling problem with sequence-dependent setup times. Picard and Queyranne (1978) provide three integer programming formulations for the time-dependent TSP. The authors analyze the relationships between the relaxations of these models by comparing their lower bounds. It is observed that the shortest path relaxation (related to the first model) is very similar to a formulation proposed by Hadley (1964) for the classical TSP. Vander Wiel and Sahinidis (1996) propose an algorithm for the time-dependent TSP, based on applying Benders decomposition to a mixed-integer linear programming formulation. The authors also develop a network-based algorithm to identify Pareto-optimal dual solutions of the highly degenerate subproblems. Results indicate that the performance

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of the algorithm is considerably improved by employing these Pareto-optimal solutions. In Bigras et al. (2008), the integer programming formulations of the time-dependent TSP are extended to solve a single machine scheduling problems with sequence-dependent setup times. Two separate objectives are considered: minimizing total flow time and minimizing total tardiness. Instances with 45 and 50 jobs can be solved exactly by the proposed branch-and-bound algorithm.

A main difference between our problem and those presented in the papers mentioned above (Bigras et al., 2008; Gouveia & Voß, 1995; Picard & Queyranne, 1978; Vander Wiel & Sahinidis, 1996) lies in the way of modeling the time-dependent component. In these papers, a discrete travel time matrix is employed, which defines the travel time along each arc with respect to the position of that arc in the tour. In other words, time-dependency is associated with the visiting order of customers, not with the arrival time or the departure time. In our problem, however, service duration is defined as a continuous function of the time at which service starts. This definition makes our problem different from the time-dependent versions studied in Bigras et al. (2008), Gouveia and Voß (1995), Picard and Queyranne (1978), Vander Wiel and Sahinidis (1996), and thus the methods proposed in these papers are not applicable to our problem.

Cordeau, Ghiani, and Guerriero (2014) consider a time-dependent TSP in which the predetermined time horizon is partitioned into a number of time intervals, and the average travel speed on each arc during each interval is known. The travel time on each arc is then computed by a procedure introduced by Ichoua, Gendreau, and Potvin (2003), and a branch-and-cut procedure is developed to solve the problem. The proposed algorithm is capable of solving instances with up to 40 nodes. In principle, a special case of our problem with linear service times can be solved by the algorithm of Cordeau et al. (2014). In that paper, travel speed functions are defined by employing the degradation of the congestion factors and the authors report that their algorithm works as long as the values of these factors are set within a given interval (approximately [0.7, 1.0]). In our case, if we consider a linear service time function, small service times occur early in the route and large service times occur towards the end. The former case translates into high degradation of congestion factors and the latter case translates into low degradation of congestion factors, which corresponds to values well outside the interval considered in Cordeau et al. (2014). Furthermore, our model can solve instances in which the FIFO property does not hold whereas this property is required for the algorithm proposed in Cordeau et al. (2014).

In terms of the service cost, Tagmouti, Gendreau, and Potvin (2007), Tagmouti, Gendreau, and Potvin (2010), Tagmouti, Gendreau, and Potvin (2011) consider time-dependency within the scope of the Capacitated Arc Routing Problem (CARP). The classical CARP aims to serve a set of required arcs at minimum cost, using a fleet of capacitated vehicles based at the depot. The three above-mentioned papers focus on a version of the CARP where each arc has a time-dependent service cost but a fixed service time. Tagmouti et al. (2007) develop a column generation algorithm, and Tagmouti et al. (2010, 2011) propose heuristics.

To the best of our knowledge, the TSP-TS has never been considered previously. In contrast to what is done in the papers just mentioned, we can handle several types of service time functions, such as linear and quadratic functions. Moreover, time-dependent service times are included into the model not only through the objective function (e.g., models with time-dependent travel times), but also through the constraints. More specifically, the service time cannot be incorporated into the arc durations.

The remainder of this paper is organized as follows. In Section 2, we describe properties of the service time function and provide the computation of a valid lower bound on the total service time of an optimal solution to our problem. In Section 3, we propose a formulation for the TSP-TS, together with three variants based on different forms of subtour elimination constraints. We also present the computation of a lower bound on the bigM which is employed in our model. Section 4 reports computational results corresponding to different subtour elimination constraints and different service time functions. This section also provides the computation of a valid upper bound on the total route duration of an optimal solution. Finally, our main findings and conclusions are highlighted in Section 5.

2. Service time function

In this section, we present the certain properties of the service time function $s_i(b_i)$ at node i.

2.1. First-In-First-Out property

The first property is related to the First-In-First-Out (FIFO) principle which states that if service at node i starts at a time b_i, any service starting at a later time b_i' at that node cannot be completed earlier than $b_i + s_i(b_i)$.

Proposition 2.1. *$s_i(b_i)$ satisfies the FIFO property if and only if* $\dfrac{ds_i(b_i)}{db_i} \geq -1$.

Proof. ($\rightarrow$ necessity)

If $s_i(b_i)$ satisfies the FIFO property, then

$$b_i + s_i(b_i) \leq b_i' + s_i(b_i'),$$

for all $b_i' > b_i$. The above statement can be rewritten as

$$s_i(b_i') - s_i(b_i) \geq -(b_i' - b_i),$$

where $b_i' = b_i + \delta$ and $\delta > 0$. The latter inequality yields

$$s_i(b_i + \delta) - s_i(b_i) \geq -\delta,$$

$$\frac{s_i(b_i + \delta) - s_i(b_i)}{\delta} \geq -1,$$

$$\lim_{\delta \to 0} \frac{s_i(b_i + \delta) - s_i(b_i)}{\delta} \geq \lim_{\delta \to 0} -1,$$

$$\frac{ds_i(b_i)}{db_i} \geq -1.$$

($\leftarrow$ sufficiency)

It is given that $s_i(b_i)$ is continuous on $[b_i, b_i']$ where $b_i' > b_i$. From the mean value theorem, we know that there exists at least one point b_i^* in (b_i, b_i') such that

$$\frac{ds_i(b_i^*)}{db_i^*} = \frac{s_i(b_i') - s_i(b_i)}{b_i' - b_i}.$$

If $\dfrac{ds_i(b_i^*)}{db_i^*} \geq -1$ for all b_i^* in (b_i, b_i'), then

$$\frac{s_i(b_i') - s_i(b_i)}{b_i' - b_i} \geq -1,$$

$$-s_i(b_i') + s_i(b_i) \leq b_i' - b_i,$$

$$b_i + s_i(b_i) \leq b_i' + s_i(b_i'),$$

which means that the FIFO property is satisfied. □

Note that Proposition 2.1 holds for any service time function. At this point, it is worth observing that a TSP solution is not always optimal for the TSP-TS, even when we apply a service time function that satisfies the FIFO property starting from the first customer in the route. To illustrate, suppose that there are three customers (denoted