European Journal of Operational Research 247 (2015) 389–400

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Perfect periodic scheduling for binary tree routing in wireless networks

Eun-Seok Kim[a], Celia A. Glass[b,*]

[a] Department of International Management and Innovation, Middlesex University Business School, London NW4 4BT, United Kingdom
[b] Cass Business School, City University London, 106 Bunhill Row, London EC1Y 8TZ, United Kingdom

ARTICLE INFO

Article history:
Received 23 January 2014
Accepted 11 May 2015
Available online 14 June 2015

Keywords:
Scheduling
OR in telecommunications
Mobile and Ad hoc NETworks (MANETs)
Combinatorial optimization
Chinese Remainder Theorem

ABSTRACT

In this paper we tackle the problem of co-ordinating transmission of data across a Wireless Mesh Network. The single task nature of mesh nodes imposes simultaneous activation of adjacent nodes during transmission. This makes the co-ordinated scheduling of local mesh node traffic with forwarded traffic across the access network to the Internet via the Gateway notoriously difficult. Moreover, with packet data the nature of the co-ordinated transmission schedule has a big impact upon both the data throughput and energy consumption. Perfect Periodic Scheduling, in which each demand is itself serviced periodically, provides a robust solution. In this paper we explore the properties of Perfect Periodic Schedules with modulo arithmetic using the Chinese Remainder Theorem. We provide a polynomial time, optimisation algorithm, when the access network routing tree has a chain or binary tree structure. Results demonstrate that energy savings and high throughput can be achieved simultaneously. The methodology is generalisable.

1. Introduction

The emerging technology of Wireless Mesh Networks (WMN) (Akyildiz, Wang, & Wang, 2005) provides a promising paradigm for the flexible and low-cost provision of global Internet communication. Mesh routers facilitate multi-hop wireless transmission to relay data over extended distances without need for the cost, delay and disruption of installing cabled access points. Packet scheduling facilitates improved throughput, fairness between clients, reduced delays and energy conservation (Quintas & Friderikos, 2012). However, specialized scheduling methodology is required to exploit these features.

Mesh routers are typically mounted on the sides of buildings and operate in two ways: firstly they service the clients who connect directly to a mesh router to gain broadband access; secondly they act as a relay to other mesh routers in forwarding content to a particular mesh router that acts as the gateway to wired infrastructure. Within each local star network the mesh router can communicate with at most one client at a time. The packet nature of transmission imposes a discrete, unit time, nature on transmission schedules. Moreover, schedules which are periodic for each client are highly desirable because they provide clients with predefined transmission times between which they can conserve resources and avoid contention. The regularity of transmission reduces jitter and thus improves Quality of Service. In addition, the issue of fairness between clients can be enforced by imposing Perfect Periodic Schedule (PPS), in which clients each have periodic sub-schedules of appropriate relative periodicity. Across a mesh network mesh routers may therefore impose local scheduling on their own clients but then need to interweave global scheduling on forwarding traffic to another mesh router. Since mesh routers are unable to multi-task, the problem of coordinating transmission across the entire routing network in the WMN is considerable. Improvement in throughput is captured by the Minimum Frame Length Schedule Problem (MFLSP) which seeks to find a schedule of minimum total duration which may then be repeated. In this paper we therefore focus on MFLSP using centrally co-ordinated periodicities to schedule packets across the network.

Several studies have been undertaken on problems of local access. Local traffic is serviced by a mesh router, and forms a local star network, each in a periodic fashion within a perfect periodic (sub)schedule. Bar-Noy, Bhatia, Naor, and Schieber (2002a) prove that the problem of finding a feasible perfect periodic schedule is an $\mathcal{NP}$-hard problem in general. Kim and Glass (2014) derive a simple test for the existence of a feasible schedule for problems with two or three distinct periodicities in total. They also provide a method of constructing a feasible schedule, if one exists, using modulo arithmetic. In practice, clients' level of requested demand may vary considerably. Due to the difficulty of finding a feasible perfect periodic schedule to satisfy the particular combination of requested periodicities, heuristics are used to allocated close values, according to specific criteria. Bar-Noy, Dreizin, and Patt-Shamir (2004) consider two

---

* Corresponding author. Tel.: +44 20 7040 8959.
E-mail address: c.a.glass@city.ac.uk (C.A. Glass).

http://dx.doi.org/10.1016/j.ejor.2015.05.031

objective measures of maximum and weighted average ratios between the allocated and requested periodicities. They present a few efficient heuristic algorithms to develop a perfect periodic schedule using a methodology, called *tree scheduling*, since it is based on hierarchical round-robin where the hierarchy is a form of tree. Bar-Noy, Nisgav, and Patt-Shamir (2002b) develop tree based approximation algorithms for perfect periodic schedule with the objective of minimizing weighted average ratios between the allocated periodicity and requested periodicity. Brakerski, Dreizin, and Patt-Shamir (2003) study the question of dispatching in a perfect periodic schedule, namely how to find the next item to schedule, given the past schedule. There are few other papers which consider PPS for telecommunications, namely (Brakerski et al., 2003; Brakerski, Nisgav, & Patt-Shamir, 2006; Chen & Huang, 2008; Patil & Garg, 2006), but none applied to WMNs.

Some studies have been undertaken on problems of data transmission across a mesh network to carry the data from individual mesh nodes to the Internet Gateway. Different interference models have been proposed in the wireless scheduling literature. Notably, the graph interference model (Commander & Pardalos, 2009; Ephremides & Truong, 1990; Gupta, Lin, & Srikant, 2007; Raman, 2006; Sarkar & Ray, 2008; Sharma, Mazumdar, & Shroff, 2006; Wang & Ansari, 1997), where nodes interfere with other nodes in a predefined neighbourhood within the network a conflict graph. If the interference is restricted to the 1-hop neighborhood, then the scheduling problem reduces to the Chromatic Number Problem. More recently the physical interference model has been proposed (Bjorklund, Varbrand, & Yuan, 2004; Brar, Blough, & Santi, 2006; Das, Marks, Arabshahi, & Gray, 2005; ElBatt & Ephremides, 2004; Goussevskaia, Oswald, & Wattenhofer, 2007; Hua & Lau, 2008; Li & Ephremides, 2007; Moscibroda, Wattenhofer, & Zollinger, 2006; Papadaki & Friderikos, 2008; Quintas & Friderikos, 2012) where signal power attenuation is taken explicitly into account via the Signal to Interference plus Noise Ratio (SINR) constraint that represents the actual physical interference in the wireless network. In the WMN context, interference related to broadcast noise is less of a feature. The main characteristic of the technology is blocking of transmission on adjacent links due to the single-task nature of mesh nodes. The problem thus resembles 1-hop edge colouring. However, the strongest feature in our context is the periodic nature of transmission through a link.

One article (Allen, Cooper, Glass, Kim, & Whitaker, 2012) explores the means of coordinating local mesh schedules which are periodic, but not necessarily so restrictive as to be perfectly periodic. The authors consider the scenario of pre-set local periodic schedules at the mesh nodes, and develop an heuristic to integrate them into a global schedule through the access network. An access link between two adjacent nodes can only be active when there is a simultaneous gap in local transmission at each of the two nodes. Thus, the first natural mechanism for co-ordinating local schedules is to control their relative start times. However, this is rarely sufficient even with sparse local schedules. Allen et al. (2012) develop an optimization scheduling algorithm which in addition equitably reduces the service time to local clients. Their algorithm works well for 25-node routing networks. However, by the nature of the problem, a large reduction in throughput was required to achieve a feasible schedule. Their computational work thus highlights the necessity of co-ordinating the periodicities of the local schedules if service levels are to be maintained. When transmission is co-ordinated in practice this necessity is satisfied with the standard mode of a Common Cycle.

We tackle the problem of scheduling both local and global data transmissions in a mesh network in perfectly periodic fashion. In a perfect periodic schedule, each transmission is undertaken at a regular, though not necessarily common, time interval.

We develop a methodology for the problem focusing upon uniform client demand, uniform link capacities and binary and chain routing trees. This is in line with the common practice of imposing routing through tree subnetworks of binary, or near binary, form. Moreover, both the results and the methodology are generalisable. Results are compared with the simpler periodic form used in practice of a Common Cycle, termed round robin, to gauge their advantage. The problem is formulated and the solution space defined in terms of congruent arithmetic in the next section. The case of a chain routing tree is then analysed in Section 3 and reduced to just two potentially optimal forms. The following three sections are dedicated to finding minimum time frame schedules for a binary routing tree. We first analyse properties of feasible, and then optimal, schedules for half of a binary tree, namely one which has (up to) two branches on all but the node adjacent to the Gateway. Using these results, in Section 5 we reduce the number of candidates for an optimal schedule of a full binary tree. The forms of an optimal binary tree are then further reduced and enumerated in Section 6, along with closed form expressions for the corresponding time frames. The outcome is an optimisation algorithm, which depends only upon prime factorisation of an integer of reasonable size, namely the total number of peripheral clients in the network. A polynomial time approximation scheme (PTAS), which is computable in practice, is also provided. The impact of transmission from different parts of the network, and the effectiveness gain over the Common Cycle schedule, are also analysed. The behaviour of algorithm OptPPS in practice is evaluated in Section 7, where experimental results reveal that efficiency gains of over 35% is normal, and 100% is reached for some relatively small networks.

2. Background

The routing of messages through a Wireless Mesh Network is done in practice within a predetermined routing tree subnetwork whose root is the single gateway to the Internet. The packet nature of data transmission results in transmissions of homogeneous size. Data all originate at local clients and in the absence of further information we assume identical demand from each client in the network.

In practice, transmission into and out of the gateway are generally performed separately. We focus upon flow into the gateway, as outflow transmission can be treated in an identical manner. In this context a mesh node may have several incoming links within the routing tree, but only a single outgoing link. It is simplest to consider the case of homogeneous link capacity, which we will calibrate to be one unit of data per time unit.

Now recall that any two links adjacent to a star-node cannot be active simultaneously. Thus, at a mesh node a schedule consists of an assignment of each time slot to at most one of the adjacent links: to a local client; to one of the incoming access links; or else the single outgoing access link. The imperative of improved throughput is captured by the Minimum Frame Length Schedule Problem (MFLSP) which seeks to find a schedule of minimum total duration. In this context, we wish to find a periodic schedule, of minimum length, in which all data make a single hop along the routing tree and each link being itself scheduled periodically. The problem may be formulated as follows.

Notations

G	index for the Gateway Mesh node
j	index for a non-Gateway Mesh node
n	number of Mesh nodes, other than the Gateway
l_j	the link in the routing tree out of Mesh node j
w_j	total amount of data flow through link j, i.e. the amount of data output by node j
$\mathcal{L}_G$	the set of links in the access network ending at the Gateway Mesh node
$\mathcal{L}_j$	the set of links in the access network ending or beginning at Mesh node j
$\mathcal{Y}_j$	the set of links from local clients into Mesh node j
y_j	$= \lvert\mathcal{Y}_j\rvert$, the number of local clients of Mesh node j