European Journal of Operational Research 247 (2015) 676–681

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Short Communication

Does mean-variance portfoliomanagement deserve expected utility's approximative affirmation?

Otto Loistl

University of Econmomics and Business, Vienna, Austria

ARTICLE INFO

Article history:
Available online 17 June 2015

Keywords:
Portfoliomanagement
Mean-variance analysis
Expected utility

ABSTRACT

This article is a comment on the paper by Harry Markowitz on "Mean-Variance approximations to expected utility", *European Journal of Operational Research*, 234 (2014), 436-355. This comment is followed by the reply of Harry Markowitz.

1. Introduction

1.1. Expected utility inspired mean variance portfoliomanagement

Harry Markowitz (2014) provides a survey of "mean-variance approximations to expected utility", "commemorating the 60th anniversary of Markowitz (1952)". But the importance of Markowitz (1952) goes far beyond this issue. His 1952 paper has been of vital importance for the modern theory in finance and for the academic and professional handling of portfolio management as well. Our comment wants to elaborate its remarkable influence on portfoliomanagement including the current issues.

Markowitz got pristine inspiration by the discussion about decision making under uncertainty/risk. See Markowitz (2015). He decided to follow Expected Utility Theory (**EUT** or short) already before it became the dominating theoretical issue. A favorite tool at the operational level was "mean variance". Markowitz based Mean-Variance-Portfoliotheory (**MVP** for short) on EUT. He feels obliged until now to proof by that connection that MVP is rational decision making procedure.

His approach intensified other discussions as well (see Markowitz, 1956): How can the huge data requirement be handled, both with regard to the data collection and the computational requirements? Sharpe (1963), referring to Markowitz publications, proposed an efficient solution, called "Diagonal Model". Substituting the entire covariance matrix by the connection with an index reduced the total number of figures required in MVP considerably. This concept has been according to Sharpe developed in close discussion with Markowitz. Sharpe (1964) referred to the "market portfolio" instead an index and created the "security market line". Later on, Merton (1972) published the proof that any efficient portfolio could serve as tangent portfolio to construct a security market line, thus giving leeway to any "index" portfolio.

The pragmatic idea of a market portfolio—renowned by its abbreviation CAPM—set a milestone in finance: The commonly used risk premium become exactly computable: The difference "market rate of return minus the riskless rate" multiplied by "beta". That term is now one of the most important in finance, both in theoretical and in professional discussions. Without Markowitz, beta would rarely exist. Nearly as important is "alpha", the riskless rate of return, or in terms of active portfolio management "portfolio managers' *benchmark-relative performance*" (Fischer & Wermers, 2013, p. V). For the different alpha definitions currently applied, see the excellent description including professional illustrations by Fischer and Wermers, Chapter 3.5.

These terms are until now at the core of professional portfoliomanagement. But none of these definitions refers to expected utility. MVP has also changed the concept of asset management regulation, e.g., ERISA (Employee Retirement Income Security Act): The vital importance of the entire portfolio's risk return tradeoff is now recognized and substituted the dominance of a single security's capital protection: "Prudent Expert" instead of "Prudent Man" rule. Active Portfoliomanagement has been developed and Portfolio Presentation incorporates MVP-tools, like "Sharpe ratio". "*The Sharpe Ratio has some very appealing benefits as a measure of portfolio efficiency. It provides a very intuitive reward to risk trade-off."* (Fischer and Wermers, 2012, p. 56.)

1.2. Approximative proposals instead of exact analysis

Another challenge was to provide evidence that the MVP-EUT connection is valid. It is still, at least in academia, an important issue. The professional day to day business developed MVP further without

DOI of original article: 10.1016/j.ejor.2012.08.023
E-mail address: otto.loistl@wu.ac.at

http://dx.doi.org/10.1016/j.ejor.2015.06.010

regarding EUT-principles. Markowitz (2014) regretfully states that his (1959) argument supporting "*mean-variance approximation to expected utility is rarely cited*" (p. 347). We may ask: What are MVP's benefits when relying on EUT?

"*Markowitz (1959) justifies mean variance analysis by relating it to theory of rational decision making over time and under uncertainty … If you believe (as many do, including me) that rational decision making should be consistent with expected utility maximization, then the necessary and sufficient condition for the use of mean-variance analysis is that a carefully selected portfolio from the mean-variance efficient set will approximately maximize expected utility, for a great variety of concave (risk-averting) utility functions*." Markowitz (2012, p. 10)

Markowitz's (2014) comprehensive survey of the MVP-EUT-investigation makes obvious the difficulties to proof the connection: Greater mathematical obstacles arise when investigating exact relations beyond some well behaving combinations of utility function and probability distribution: Chamberlain (1983) shows, *"if there is a riskless asset, then the distribution of every portfolio is determined by its mean and variance if and only if the random returns are a linear transformation of a spherically distributed random vector*. (p. 185).[1] Bringing "spherically distributed random variables" down to an operational level, one may arrive at the well known "normal distribution". But even in this case, the exact EUT-solution is not easy to handle when the normal distribution is combined with an e.g. logarithmic utility function. Markowitz (2014) survey shows that many scholars obviously choose pragmatically a different mode of reasoning: Preferring **approximative** solutions for **practical purposes.**[2]

Acceptable approximations are advocated pragmatically: "*Levy and Markowitz (1979) find that mean-variance approximations are* ***usually quite accurate****.*" Markowitz (2014, p. 374). He tries to incapacitate Loistl's (1976) analytically exact results applying Taylor's Series Expansion (**TSE** for short) by the assertion of an "erroneous analysis".

Markowitz harsh assertion is inadequate: Since both the analytical arguments and their numeric illustrations as well are correct, nothing is wrong in Loistl's paper. Only one typo has to be mentioned: In formula 7b the exponential utility function's coefficient *b* amount in the numerical example 0.05 instead of the printed 0.5. But it is given correctly in FN 4. Anyway, Markowitz' assertion does not refer to this typo. Furthermore, Loistl's (1976) complete remark indicates that it is not really a contradiction to Markowitz, especially not regarding his "approximative" attitude. "*Looking at the figures, we are forced to an ambivalent conclusion: the mean-variance approximation is not a good approximation of the expected utility at all; however, it is more exact than a Taylor's series expansion including higher terms of any order*" (Loistl, 1976, p. 909). Why then tried Markowitz to condemn the exact TSE result by insufficient accusation?

Loistl (1976) is a caveat against the careless application of TSE in case of infinite limits. But Markowitz (2014) did not deal with TSE problems explicitly. Why spend so much efforts for an issue that is neglected by professionals and is treated by approximative measures quite (un)satisfactorily? The common procedures in every day professional portfolio management do not refer to EUT. See Section 3.

2. Soft assumptions and approximative acceptable solutions

2.1. Difficulties maximizing expected utility

Markowitz (2012) concedes "*Typically it is much more convenient and economical to determine the set of mean variance efficient portfolios than it is to find the portfolio which maximizes expected utility. … It still takes many times as long as to compute the expected value of most concave functions as it does to trace out a mean variance efficient frontier….. Levy and Markowitz (1979) conclude, for some hypothetical investor Mr. X, that if Mr. X can carefully pick the MV efficient portfolio which is best for him, then Mr. X, who still does not know his current utility function, has nevertheless selected a portfolio with maximum or almost maximum expected utility.*" Markowitz (2012, p. 14). He realizes the difficulties "*when one explicitly maximizes expected utility. One needs to determine what type of joint probability distribution generates returns combinations… and must estimate the parameters for such a joint distribution*". Furthermore, *"someone must determine the investor's utility function…. Finally, another advantage of using implicit EU maximization is that no one has to explain the expected utility concept to the investor or to the supervisory board of an institutional investor, or to the typical financial advisor. Instead, portfolio choice can be couched in the familiar terms of risk versus returns."*(p. 15). Regarding the disclosure obligations required by e.g. ERISA and SIP (statement of Investment Principles/Policy), such a scenario might be quite optimistic. See e.g. Loistl and Petrag (2006, p. 293) and Fischer and Wermers (2012).

Markowitz (2014) refers to "***approximative***" statements and his goal is to establish "***sufficient*** *conditions for the use of mean-variance analysis in* ***practice***" (p. 346). But not in the mathematical sense of a proof. He believes that…"*a* ***careful choice*** *from a mean variance efficient frontier will* ***approximately*** *maximize expected utility for a* ***wide variety*** *of concave (risk-averse) utility functions*" (p. 346).[3]

Loistl (1976) was instigated by Samuelson's (1970) article; That ignored the pitfalls of infinite limits when applying TSE. Loistl (1976) was not dealing with Markowitz (1959) and the obviously approximative statements. Imprecise terms like "*in practice*", "*practical use,*" "*careful choice*", "*approximately*" make it difficult to check statements relying on such terms.

2.2. Inconsistent numerical examples vs analytical arguments

The reverse is true too: An analytical error has to be verified exactly. But Markowitz made only a perfunctory remark; and even that is misleading.

Markowitz (2014) taunts Loistl for confusing percentage return and absolute return figures. He illustrates this allegation by the example that a 600 percent return is confused with a 6 percent return and thus an end of period wealth of 7000.000 instead of 1060.000 is erroneously calculated (p. 349). But he explains nowhere in his article where Loistl (1976) committed such a fault, furthermore why such a fictitious faulty calculation justifies to disqualify the analysis "erroneous", because there is no such a miscalculation.

From this allegation, Markowitz (2014) deduces obviously, that Loistl has mixed 30 with 0.30: He concludes, Loistl's "erroneous" TSE results are due to that wrong scaling and cites his 1959 study: "*a quadratic fits well for returns in the interval* -0.30 *to* 0.40" and infers from that: *"Since a 30 percent gain must be represented by* $R = 0.30$ *rather than* $R = 30$*….Loistl's negative conclusion about the mean-variance approximation is due to an erroneous analysis rather than an erroneous approximation"* (p. 349).

Markowitz implicitly assumes obviously, that Loistl's TSE-results would exhibit a "better fit" when the calculation would have been made with figures like -0.30 and 0.40 instead of -30 and 40. This argument is incorrect because incomplete: Investigating the TSE fit of figures like -0.30 and 0.40 instead of -30 and 40 has to regard the complete TSE-scenario: If TSE is expanded around a value of 0.25 in the first case and around 25 in the second, than the approximation quality will be in both cases similar. If TSE is expanded in both cases

[1] Markowitz (2014) quotes Chamberlain as requiring a distribution "joint elliptical", for EUT being a function of mean and variance. But Chamberlain refers throughout his paper to spherically distributed random vector. The difference/similarity between the terms is not explained.

[2] Bold letters added by OL.

[3] Bold letters added by OL.