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Discrete Optimization

A minimum cost network flow model for the maximum covering and patrol routing problem

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ABSTRACT

This paper shows how the maximum covering and patrol routing problem (MCPRP) can be modeled as a minimum cost network flow problem (MCNFP). Based on the MCNFP model, all available benchmark instances of the MCPRP can be solved to optimality in less than 0.4s per instance. It is furthermore shown that several practical additions to the MCPRP, such as different start and end locations of patrol cars and overlapping shift durations can be modeled by a multi-commodity minimum cost network flow model and solved to optimality in acceptable computational times given the sizes of practical instances.

1. Introduction

The maximum covering and patrol routing problem (MCPRP) was introduced by Keskin, Li, Steil, and Spiller (2012) and is used to assist traffic enforcement. A typical method for state troopers is to patrol "hot spots" which are certain locations on highways where particular types of crashes (e.g. crashes caused by speed or driving under influence) frequently occur (Anderson, 2007; Steil & Parrish, 2009). Furthermore, these hot spots are only active during certain time windows. Due to limited resources, not all active hot spots can be patrolled. Therefore, an optimization problem to route patrol cars in a way that maximizes hot spot coverage appears to be appropriate. Keskin et al. (2012) model the MCPRP as a variant of the orienteering problem Tsiligirides (1984); Vansteenwegen, Souffriau, and Van Oudheusden (2011), prove that their model is NP-hard, and present two heuristics, a local search heuristic and a tabu search heuristic which determine good quality solutions in short periods of time. The authors claim that a heuristic solution instead of an exact technique is preferred for their model since it is important for the practitioner to obtain a good solution quickly.

Having an efficient and effective solution method to solve the MCPRP is useful since the problem often appears as a sub problem in larger problems. Li and Keskin (2013) consider a bi-objective multi-period patrol routing problem. The multi-period aspect appears through the introduction of intermediate temporary stations in the patrol routes. Li and Keskin develop a heuristic that exploits the hierarchical structure of the problem by decomposing the problem in a location and a routing problem. An effective solution method for the MCPRP can be incorporated in such a framework to solve the routing problem more efficiently. If an orienteering problem approach is taken, this is similar to the orienteering problem with hotel selection (OPHS) of Divsalar, Vansteenwegen, and Cattrysse (2013); Divsalar, Vansteenwegen, Sörensen, and Cattrysse (2014) which considers a multi-period tourist trip planner application. Very recently, Çapar, Keskin, and Rubin (2015) reconsidered the MIP formulation for the MCPRP and also used a set of domination rules to greatly simplify the MIP formulation. The new MIP formulation is able to solve their set of benchmark instances to optimality within reasonable calculation times. It also allows several extensions to be tested, such as, letting troopers start from their homes, allowing delayed starts and intraday diversion.

Other patrol routing problems that require rerouting when incidents in the network occur during the execution of a patrol routing scheme are considered by Moonen, Cattrysse, Oudheusden, (2008), Takamiya and Watanabe (2011), Chen (2012), and Portugal and Rocha (2013). These problems can possibly also benefit from a fast solution method for rerouting patrol cars.

Modeling the MCPRP as a minimum cost network flow problem (MCNFP) (Winston, 1987), which is known to be solvable in polynomial time (Orlin, 1997) would be a major improvement over existing models in the literature. This is an example of exploiting the network structure of a problem to gain computational efficiency and it underscores the importance of selecting an appropriate model for a particular problem. In this particular case, the network structure is uncovered in the following sections step by step. For problems where the network structure is not obvious, a systematic approach such as the

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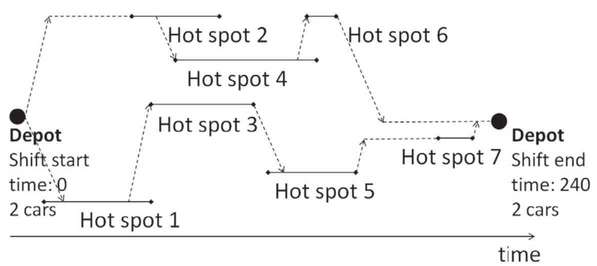

Fig. 1. The maximum covering and patrol routing problem with two cars.

netform concept described by Glover, Klingman, and Phillips (1990) can be used to explicit this structure.

Several other patrol routing applications, each with their own specific constraints can be found in the literature. Some of the more recent ones are Lou, Yin, and Lawphongpanich (2011), Willemse and Joubert (2011), and Chircop, Surendonk, van den Briel, and Walsh (2013). These applications differ substantially from the basic MCPRP as described in Keskin et al. (2012) which prohibits a straightforward application of the proposed MCNFP reformulation. Because of the binary nature of the "hot spot profits", these applications share more characteristics with the orienteering problem or the rural postman problem (Eiselt, Gendreau, Laporte, & Laport, 1995) than with the MCNFP.

The remainder of this paper is structured as follows. In Section 2, we show how the original MCPRP problem can be modeled as an MCNFP. In Section 3, we discuss the results of the computational experiments applied on the instances of Keskin et al. (2012). In Section 4, we show how practical extensions to the MCPRP can be modeled by adding a multi-commodity aspect to the MCNFP (Tomlin, 1966) and discuss the results of some computational experiments on artificial data sets in Section 5. Finally, in Section 6, we provide our conclusions and recommendations for future research.

2. Problem analysis

Given a set of patrol cars and a set of hot spots, the objective of the MCPRP consists of finding a set of routes for the patrol cars that maximizes the time spent in hot spot locations. The travel times between hot spots are constant and known beforehand. Each hot spot is only active during certain time windows. All cars have the same shift start and end times and the same start and end locations. In addition, multiple cars being present in the same hot spot at the same time does not increase the objective function (Keskin et al., 2012). A car can enter and leave a hot spot at any given time but only collects *"gain"* for the duration that a car remains in the hot spot within the hot spot's time window. Fig. 1 represents a possible routing of two patrol cars in a graphical way. Both patrol cars are allowed to leave the depot at time 0 and need to return at the end of their shift to the depot. The horizontal dotted lines represent the fact that a car can arrive at a hot spot before the hot spot's time window is open. This time does not provide any *gain* and is called *dead time*. The angled dotted lines represent the actual movement of the cars between hot spots. In the basic MCPRP, when making abstraction of the underlying road network, the travel times between hot spots are assumed to be constant and subject to the triangle inequality. Additionally, the gain per minute can be different for each hot spot but is assumed to be constant for the full duration of the hot spot's time window.

2.1. Domination rule

The transformation of the MCPRP into a minimum cost network flow problem relies on the fact that a lot of solutions of the MCPRP are dominated. In an optimal solution, a patrol car will always stay at its current hot spot until the end of its time window unless another hot spot becomes available earlier and a larger *gain* can be obtained in the latter hot spot, taking the travel time between hot spots into account.

Thus, patrol cars will enter a hot spot i either at the opening of its time window ($t_{o,\,i}$) or at the closing time of another hot spot j's time window plus the travel time between both hot spots, denoted as $t_{a,\,ji}$. Similarly, a hot spot i will only be exited at the closing of its time window ($t_{c,\,i}$) or at the opening of another hot spot j's time window minus the travel time between both hot spots, denoted as $t_{b,\,ij}$. This results in splitting each hot spot's time window i into segments, referred to as *time sections*. The start and end points of the time sections of a hot spot i are defined by $t_{o,\,i}$, the $t_{b,\,ij}$'s, the $t_{a,\,ji}$'s and $t_{c,\,i}$. From this point on, for the sake of brevity, we will refer to a time section as a section.

To further clarify the process of identifying sections required to optimize the problem, consider the example shown in Fig. 2(a). The network consists of three hot spots with overlapping time windows. In order to identify the sections, a forward and a backward pass over all hot spots is executed. Fig. 2(b) shows the forward pass. From every hot spot's end time (including the source), the arrival time at any other hot spot (including the sink) is determined, or in other words, all $t_{a,\,ji}$'s are determined. This process is depicted by the dotted lines. The gray lines mean that the end time of the previous hot spot's time window plus the travel time is smaller than the start time of the next hot spot's time window. Likewise, the red dotted lines mean that the possible arrival time in the next hot spot falls outside the hot spot's time window. As a consequence, these "dotted" relationships will not create additional sections. However, the green dotted lines emanating from hot spot *2* will cause splits in the time windows of hot spots *1* and *3*. Fig. 2(c) shows the backward pass which determines all $t_{b,\,ij}$'s. Ultimately, Fig. 2(d) shows all identified sections.

To reiterate, we define a section as a time segment of the hot spot's time window. It is characterized by the fact that, in an optimal solution, a car can only enter a section at the start of the section's time window and only leave it at the end of the section's time window. We formulate this domination rule as the following lemma:

Lemma 1. *Any routing where a car moves from a section S before the end time of the section to another section T will be dominated by either the solution where the car leaves at the end time of section S or the solution where the car did not enter section S and immediately entered section T.*

A proof of this lemma can be found in Appendix A

Without loss of generality, the patrol routing problem is then redefined as: "Find a set of routes for the patrol cars that maximizes the time spent (or *gain*) at the sections under the additional constraint that a section can only be visited by at most one car. It should be noted, however, that it is still physically possible for two patrol cars to be present at the same hot spot location at the same time. However, this implies that one of the cars is waiting to move to another hot spot or arrived too early at the current hot spot. This physical situation will be modeled by one car actually visiting the section, collecting the gain, and one car using a travel arc that physically passes by this section while moving from or to another hot spot, not collecting the gain and thus not visiting the section.

We will now first define the general Minimum Cost Network Flow Problem (MCNFP) and then we will explain how these "sections" can be used to model the Maximum Covering Patrol Routing Problem (MCPRP) as an MCNFP.

2.2. Minimum cost network flow problem

The Minimum Cost Network Flow Problem consists of finding the cheapest possible way of sending a given amount of flow through a network, where a cost and capacity is associated with each arc in the network. It can be modeled by the following linear program, where