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Optimal loading of system with random repair time

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ABSTRACT

This paper considers single-component repairable systems supporting different levels of workloads and subject to random repair times. The mission is successful if the system can perform a specified amount of work within the maximum allowed mission time. The system can work with different load levels, each corresponding to different productivity, time-to-failure distribution, and per time unit operation cost. A numerical algorithm is first suggested to evaluate mission success probability and conditional expected cost of a successful mission for the considered repairable system. The load optimization problem is then formulated and solved for finding the system load level that minimizes the expected mission cost subject to providing a desired level of the mission success probability. Examples with discrete and continuous load variation are provided to illustrate the proposed methodology. Effects of repair efficiency, repair time distribution, and maximum allowed time on the mission reliability and cost are also investigated through the examples.

1. Introduction

Empirical studies have demonstrated that the workload applied to a system component can have a great impact on the component's failure behavior, productivity, as well as operation cost (Amari, Misra & Pham, 2008; Iyer & Rosetti, 1986; Kapur & Lamberson, 1977). Consequently, the success probability and cost of mission involving a specified amount of work to be accomplished by the system components are also dependent on the component load levels. Therefore, for systems supporting different levels of component workloads, the optimal loading problem arises where the expected mission cost is minimized subject to providing a certain level of system reliability. In this paper, we solve the optimal loading problem for repairable systems with a single component that can function at different loads or productivity levels and is subject to a random repair time every time it fails.

A repairable system is a system that can be restored to some fully satisfactory performance through maintenance actions (e.g., part adjustment or replacement) after failing to function correctly (Ascher & Feingold, 1984; Lindqvist, 2006; Yang, Zhang, & Hong, 2013). Based on the degree to which the operating condition of a system is restored through the maintenance, three categories or levels of repair models can be identified: perfect repair, general/imperfect repair, and minimal repair (Lindqvist, 2006; Yañez, Joglar, & Modarres, 2002). A perfect repair restores a system to an "as good as new" condition; a minimal repair restores the system to an "as bad as old" condition, i.e., to the same state as it was immediately before its failure; a general repair can bring the system to any condition between the former two cases.

Considerable research efforts have been dedicated to analyzing and optimizing single-component and multi-component repairable systems under those different repair models. The commonly used techniques for modeling the failure process or reliability of a repairable system include, for example, renewal processes (particularly homogeneous and non-homogeneous Poisson processes) (Saldanha, de Simone, & Frutuoso e Melo, 2001; Weckman, Shell, & Marvel, 2001), Markov chains (Bloch-Mercier, 2001, 2002; Marquez & Heguedas, 2002; Soro, Nourelfath, & Aït-Kadi, 2010), geometric processes (Castro & Pérez-Ocón, 2006; Jia & Wu, 2009; Lam, 1988; Zhang & Wang, 2007), and Bayesian methods (Percy, 2002; Rosqvist, 2000; Sheu, Yeh, Lin, & Juang, 2001). Different types of optimization problems have also been formulated and solved for repairable systems with different maintenance policies or behaviors. For example, the replacement policy optimization problems have been addressed for repairable systems subject to waiting repair times (Jia & Wu, 2009a), with repairman who can take multiple vacations (Jia & Wu, 2009b; Yuan & Xu, 2011), or under free-repair warranty (Yeh, Chen, & Lin, 2007). The optimal periodic and non-periodic inspection scheme problems have been solved for repairable systems subject to hidden failures or interactions between hard and soft

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Nomenclature

W	total amount of work in the mission
N	maximal possible number of repairs during the mission
τ	maximum allowed mission time
R	mission success probability
C	conditional expected cost of successful mission
θ_j	conditional expected mission time given that the mission succeeds after j repairs
$\langle T_j, X_j\rangle$	event when the jth failure happens at time T_j and the system spends time X_j in operation mode before the failure
$Q_j(t,x)$	joint distribution function of random values T_j and X_j
$q_j(v,w)$	probability that jth failure happens in time interval v and the system spent in operation mode w intervals before the failure
D	random repair time for system
$d_{\min}$, $d_{\max}$	minimal and maximal possible realizations of D
g_L	productivity of system working with load level L
$f_L(t)$	*pdf* of time-to-failure distribution of system working with load level L
$F_L(t)$	*cdf* of time-to-failure distribution of system working with load level L
$\psi(t)$	*pdf* of repair time
$\Psi(t)$	*cdf* of repair time
m	number of discrete intervals considered in the numerical algorithm
Δ	duration of a discrete time interval: $\Delta = \tau/m$
h	number of time intervals needed to complete the mission if no failures occur
$\lfloor x\rfloor$	floor operation that returns the maximal integer not exceeding x
Y_j	event when the mission is completed after j repairs
r_j	$\Pr(Y_j)$
η_L, β_L	scale, shape parameters of Weibull time-to-failure distribution for load level L
c_r	per time unit repair cost
$c_o(L)$	per time unit operation cost under load level L
z	repair efficiency coefficient

failures (Golmakani & Moakedi, 2012a, 2012b, 2013; Taghipour & Banjevic, 2011, 2012, Taghipour, Banjevic, & Jardine, 2010). In (Yeh & Lo, 2001), a joint optimization problem has been solved for repairable systems to determine the optimal number and schedule of preventive maintenance actions as well as the corresponding maintenance degrees. In (Monga & Zuo, 1998), the optimal maintenance and warranty policy have been addressed for repairable systems considering all phases of the system life cycle, where the optimal burn-in period, optimal preventive maintenance intervals and optimal replacement times are determined.

Despite the rich literature on modeling and optimization of repairable systems, to the best of our knowledge, none of those works have considered the optimal loading problem. As indicated at the beginning of this section, solutions to the optimal loading problem can facilitate planning of a mission operation with the minimum expected mission cost while satisfying a certain level of mission success probability. In this work we make the novel contributions by formulating and solving the component load optimization problem for single-component repairable systems, which support multiple different levels of workloads and have random failure and repair times. The proposed methodology is flexible and general to accommodate different levels of repair models (perfect, imperfect, and minimal) as well as arbitrary types of component time-to-failure and repair time distributions.

The reminder of the paper is organized as follows. Section 2 describes the system model as well as the load-failure and load-productivity relationship models. The formulation of the load optimization problem is also given. Section 3 presents the evaluation of mission success probability and conditional expected cost of successful mission for single-component repairable systems subject to random repair times. Section 4 presents the numerical evaluation algorithm. Section 5 presents two illustrative examples. Impacts of repair efficiency, repair time distribution parameters, and the maximum allowed mission time on the mission success probability and conditional expected cost are investigated. Section 6 concludes the paper and gives directions of future work.

2. The problem description

2.1. The model

The system should perform an amount of work W within a fixed mission time τ. It can work with different load levels. Each load level L corresponds to a different system productivity g_L and time-to-failure distribution determined by the cumulative distribution function (*cdf*) $F_L(t)$. When the system fails, the repair/replacement procedure starts immediately. The repair time depends on external factors such as availability and efficiency of the repair manpower and equipment and is randomly distributed in the interval $[d_{\min}, d_{\max}]$. *cdf* $\Psi(t)$ of the repair time distribution is known. We assume that if the system has operated time t_0 before the repair, its time-to-failure distribution after the repair has *cdf* $F_L(zt_0+t)$, where z is the repair efficiency coefficient that can vary from 0 (the system after repair is as good as new, which corresponds to the replacement by a brand new system or perfect repair) to 1 (the system is as bad as old, which corresponds to the minimal repair). The per time unit repair cost of the system c_r is fixed. The per time unit operation cost of the system $c_o(L)$ depends on the load level L.

2.2. Modeling load-failure and load-productivity relationship

There exist different models to represent the relationship between the load and the failure behavior of a system, such as the accelerated failure-time model (AFTM) (Levitin & Amari, 2009; Nelson, 1990; Pike, 1966) and proportional-hazard model (PHM) (Cox, 1972; Kumar & Klefsjo, 1994). For example, in the widely applied AFTM model the effect of the system load is multiplicative in time and the reliability and failure functions for a system with load L can be respectively expressed as $R_L(t) = R_0(t\phi(L))$, $F_L(t) = F_0(t\phi(L))$, where $R_0(.)$ and $F_0(.)$ are the baseline reliability and failure functions of the system. Two common forms of the load factor function $\varphi(L)$ have been used: $\phi(L) = L^{\alpha}$ (referred to as the power law) and $\phi(L) = e^{L\alpha}$ (referred to as the exponential law) (Levitin & Amari, 2009). In this work we adopt the most general case where the failure function, i.e., *cdf* of the time-to-failure distribution $F_L(t)$ can be different for a different load level L.

The performance or productivity of a system g_L is also dependent on the applied load L. In general, g_L is considered as a function that maps the load to the corresponding productivity. The function can take any form (Levitin & Amari, 2009). For example, g_L can be a linear function for a pipe carrying fluid in a laminar flow mode where the productivity, i.e., throughput of the pipe is proportional to the load (i.e., pressure) on the pipe. g_L can appear as a nonlinear function for a pipe carrying fluid in a turbulent flow mode where the pressure on the pipe and the volume of the flow exhibit non-linear relationship.

We assume that for any load level L, the time W/g_L needed to complete the mission is less than the maximal allowed mission time τ. The productivity that does not meet this requirement cannot provide