European Journal of Operational Research 246 (2015) 140–153

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

A noisy principal component analysis for forward rate curves

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ARTICLE INFO

Article history:
Received 6 May 2014
Accepted 21 April 2015
Available online 27 April 2015

Keywords:
Finance
Pricing
Principal component analysis
Term-structure of interest rates
HJM models

ABSTRACT

Principal Component Analysis (PCA) is the most common nonparametric method for estimating the volatility structure of Gaussian interest rate models. One major difficulty in the estimation of these models is the fact that forward rate curves are not directly observable from the market so that non-trivial observational errors arise in any statistical analysis. In this work, we point out that the classical PCA analysis is not suitable for estimating factors of forward rate curves due to the presence of measurement errors induced by market microstructure effects and numerical interpolation. Our analysis indicates that the PCA based on the long-run covariance matrix is capable to extract the true covariance structure of the forward rate curves in the presence of observational errors. Moreover, it provides a significant reduction in the pricing errors due to noisy data typically found in forward rate curves.

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1. Introduction

The term-structure of interest rates is a high-dimensional object which has been the subject of much research in the finance literature. It is the natural starting point for pricing fixed-income securities and other financial assets. In particular, the identification of factors capable to explain its movements plays a crucial role in modeling complex interest rate derivative products. Since the seminal works of Steeley (1990), Stambaugh (1988) and Litterman and Scheinkman (1991), it is well-known that most of the covariance yield curve structure can be summarized by just a few unobservable factors. This stylized fact is fundamentally based on the Principal Component Analysis (henceforth abbreviated by PCA) based on sample covariance matrices. In this case, a small number of eigenvectors summarize the whole second moment structure of the yield curves.

The interest rate markets can be summarized by two fundamental high dimensional objects: the yield $x \mapsto y_t(x)$ and forward rate curves $x \mapsto r_t(x)$; $t \geq 0$ which are connected by the following linear relation:

$$y_t(x) = \frac{1}{x}\int_0^x r_t(z)dz; \quad 0 \leq t < \infty, \quad x \geq 0. \tag{1.1}$$

See e.g. Filipovic (2009) for more details. In particular, the underlying covariance structure of yield and forward rate curves play a major role in the statistical analysis of the term-structure of interest rates. See e.g Rebonato (2002), Schmidt (2011) and other references therein. For instance, forward rate curves play a central role in pricing and hedging interest rate derivatives by means of the classical methodology proposed by Heath, Jarrow, and Morton (1992). Their contribution can be summarized by the representation of the forward rate curve dynamics in terms of a stochastic partial differential equation

$$dr_t(x) = \left(\frac{\partial r_t(x)}{\partial x} + \alpha_{HJM}(t, r_t(x))\right)dt + \sum_{j=1}^{d}\sigma^j(t, r_t(x))dB^j_t; \quad 0 \leq t < \infty, \quad x \geq 0, \tag{1.2}$$

where $r_0(x) = \xi(x)$; $x \geq 0$ is a given initial forward rate curve, α_{HJM} is the so-called (HJM) drift condition which is fully determined by the volatility structure $\sigma = (\sigma^1, \ldots, \sigma^d)$ and $(B^1, \ldots, B^d)$ is a d-dimensional Brownian motion. Indeed, the initial forward rate curve ξ and the volatility structure σ fully determine the no-arbitrage dynamics of the model (1.2). In the remainder of this paper, the family of models of the form (1.2) parameterized by volatilities σ will be called HJM models. See e.g Filipovic (2009) for further details.

For a given initial forward rate curve, the fundamental object which encodes the whole dynamics of (1.2) is volatility. In particular, due to closed form expressions for derivative prices and hedging, it is common (see e.g. (Falini, 2010; Jarrow, 2002; Rutkowski, 1996)) to assume that the volatility structure is deterministic. In this case, the stochastic dynamics of forward rates is given by a Gaussian HJM model:

$$dr_t(x) = \left(\frac{\partial r_t(x)}{\partial x} + \alpha_{HJM}(x)\right)dt + \sum_{j=1}^{d}\sigma_j(x)dB^j_t; \quad 0 \leq t < \infty, \quad x \geq 0. \tag{1.3}$$

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http://dx.doi.org/10.1016/j.ejor.2015.04.038
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The most common alternative to estimate the underlying volatility structure is to use PCA methodology (see e.g. Filipovic, 2009; Jarrow, 2002; Schmidt, 2011 and other references therein) based on the static covariance matrix of a given sample $(r_t(x_1), \ldots, r_t(x_M))$. The PCA methodology provides the following estimator for the volatility structure:

$$\hat{\sigma}^i = \hat{\varphi}_i \sqrt{\hat{\lambda}_i}; i = 1, \ldots, \hat{m}, \tag{1.4}$$

where $\hat{m}$ is the estimated number of principal components of the forward rate curves and the estimated eigenvalues and eigenvectors of the associated static covariance matrix are given by $\hat{\lambda}_i$ and $\hat{\varphi}_i$, respectively. See also Bermin (2014) for a recent theoretical link between PCA based on conditional covariance matrices and the Brownian dimension d which drives forward rates curve processes (1.2).

There are many drawbacks when estimating parameters and structures related to forward rate curves: (a) The intrinsic infinite-dimensionality of forward rate markets (see e.g Carmona and Tehranchi, 2006), (b) daily calibration may lead to inconsistent conclusions due to a possible non-existence of finite-dimensional realizations produced by (1.2) (see e.g Filipovic, 2001), and (c) the absence of fully observed forward rate curves in interest rate markets (see e.g. McCullogh, 1971). In this article, we discuss three fundamental issues related to (c) and the common use of PCA methodology in forward rate markets: Evidence of observational errors in forward rate markets; the sensitiveness of the PCA methodology facing noisy forward rate curves and their implications to pricing interest rate derivatives.

The next section continues with a brief literature review about the use of PCA methodology in interest rate data. In Section 3, we precisely state the main questions we want to tackle in this article. In Section 4, we report an elementary result about the equivalence of ranks between the covariance operators of the forward rate and yield curves. In Section 5, we describe some alternatives of estimating covariance structures in the presence of observational errors, the so-called LRCM estimators. Section 6 presents a detailed simulation analysis reporting the performance of the PCA based on LRCM estimators. Moreover, we discuss the role played by measurement errors in the PCA methodology applied to the term-structure of interest rate. In order to compare the simulation results reported in Section 6 with a real data set, Section 7 provides an empirical analysis on the number of principal components for US and UK term-structure of interest-rates. In Section 8, we analyze the impact of neglecting observational errors in pricing interest rate derivatives in light of the PCA methodology. Section 9 presents the final remarks.

2. Literature review

The literature on the PCA methodology in interest rate markets is vast. This section reviews work from a stream of this literature that has some relevance to our study.

2.1. PCA-based estimation for covariance structures in forward rate markets

Plenty of spot interest rate data (and hence yield curve data) are available in fixed income markets. However, due to the absence of explicit forward rate markets, implied forward rate curves have to be estimated from interest rates based on other financial instruments (see e.g. McCullogh, 1971). This already presents a major difficulty in implementing derivative pricing models based on the classical Heath-Jarrow-Morton methodology. See e.g. Bermin (2014); Chiu, Fang, Lavery, Lin, and Wang (2008); Filipovic (2009); Richter and Teichmann (2014) and other references therein for further details.

The most common non-parametric procedure for estimating the covariance structure of forward rate curves is the PCA methodology. Basically, three common strategies are very popular among practitioners in the PCA estimation of the forward rate curves: **(A)** One postulates the existence of a finite-dimensional parameterized family of smooth curves $\mathcal{G} = \{G(z; x); z \in \mathcal{Z} \subset \mathbb{R}^N, x \geq 0\}$ and a $\mathcal{Z}$-valued state process Y such that

$$y_t(x) = G(Y_t; x) \quad \text{for} \quad x \geq 0, \quad 0 \leq t < \infty. \tag{2.1}$$

By interpolating the available yield data based on $\mathcal{G}$, then one extracts the associated forward rate curve by means of any numerical scheme to recover $r_t(x) = y_t(x) - x\frac{\partial y_t(x)}{\partial x}$. The PCA is then applied on this estimated forward rate curves, as discussed in e.g. Jarrow (2002) and Lord and Pelsser (2007). **(B)** Instead of (2.1), one shall use a non-parametric polynomial splines method to interpolate the yield data and userelation (1.1) to recover $x \mapsto r_t(x)$ at some time $t \geq 0$. See e.g. Vasicek and Fong (1982), Barzanti and Corradi (1998), Chiu et al. (2008) and other references therein for further details. Alternatively, one can use proxies to construct the forward rate curve jointly with a given interpolating family of smooth curves $\mathcal{G}$. See e.g. Bhar, Chiarella, and Tô (2002), Alexander and Lvov (2003) and Gauthier and Simonato (2012) for further details.

One fundamental assumption behind all the classical aforementioned procedures and, more generally, on the use of PCA methodology in data analysis is the following one:

Assumption (I) There is no observational errors in forward rate curves.

2.2. Observational errors and principal component analysis

One can argue that assumption **(I)** is not too strong due to the linear relation (1.1). Apparently, mild assumptions on the parametric form $\mathcal{G}$ would alleviate a possible violation of assumption **(I)** in forward rate markets. However, caution is advised in this regard, because the marginal nature of the forward rate curves encoded by $\frac{\partial y_t(x)}{\partial x}$ may introduce a severe bias. One of the main issues in calibration of HJM models is the fact that arbitrary choices of parametric forms $\mathcal{G}$ for $x \mapsto y_t(x)$ may lead to inconsistency of the estimates over a trading period (see e.g. Filipovic, 2001 and other references therein). Moreover, proxies may cause non-negligible observational errors which might contribute to the so-called microstructure effects (see e.g. Mizrach & Neely, 2011 and Goyenko & Ukhov, 2011). In fact, the presence of measurement errors may cause biased and inconsistent parameter estimates. This may lead to erroneous conclusions to various degrees in the financial analysis.

At this stage, a natural question is the validity of assumption **(I)** in the term-structure of interest rates. In fact, we shall compare the existing literature of principal components between yield and forward rate curves to see some evidence of violation of assumption **(I)**. In one hand, the linearity of the relation (1.1) strongly suggests that the "dimension" of the forward rate and yield curves must be identical (See Proposition 4.1). On the other hand, distinct results in the literature have been reported on the spectral structure of the forward rate and yield curves. Akahori, Aoki, and Nagata (2006) and Liu (2010) report a remarkable difference in the estimated number of factors between forward rates and yield curves and they suggest that a possible explanation for this would be the violation of the random walk hypothesis. The same type of behavior was reported by Lekkos (2000) who argues that averaging the forward rates over time to maturities would induce a strong dependence on the yield data. He argues that the PCA method artificially estimates a small number of principal components for yield curves. Alexander and Lvov (2003) study statistical properties of the UK Libor rates. They show that the strategy **(A)** induces equivalent loading factor structures between yield and implied forward rate curves. Lord and Pelsser (2007) report a visible difference in the PCA of forward and yield curves by using estimated Svensson curves for the Euro market. Similar results have been reported by

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