European Journal of Operational Research 246 (2015) 281–292

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Optimal firm growth under the threat of entry[☆]

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ARTICLE INFO

Article history:
Received 18 February 2014
Accepted 17 April 2015
Available online 24 April 2015

JEL Classfication:
C73
D92
L13

Keywords:
Economics
Game theory
Dynamic programming

ABSTRACT

The paper studies the incumbent-entrant problem in a fully dynamic setting. We find that under an open-loop information structure the incumbent anticipates entry by overinvesting, whereas in the Markov perfect equilibrium the incumbent slightly underinvests in the period before the entry. The entry cost level where entry accommodation passes into entry deterrence is lower in the Markov perfect equilibrium. Further we find that the incumbent's capital stock level needed to deter entry is hump shaped as a function of the entry time, whereas the corresponding entry cost, where the entrant is indifferent between entry and non-entry, is U-shaped.

1. Introduction

The paper studies the incumbent-entrant problem in a fully dynamic setting. Initially the incumbent offers a homogenous product. To increase production capacity the firm can invest to enlarge its capital stock. From some given future point in time on, another firm can enter this market. In case entry takes place, a duopoly market arises with homogenous products. The question is how the incumbent invests in order to anticipate the future entry threat. Basically, it can choose between a policy of entry deterrence and entry accommodation. In the latter case we also investigate what happens after the second firm has entered. Then two firms are in the market and both can invest to increase production capacity.

First, we consider a situation where at some given future point in time an inevitable entry takes place. This allows us to establish the optimal entry accommodation policy.

However, under a Markov perfect equilibrium information structure the incumbent slightly underinvests in the period before entry takes place. The reason is that in such a framework a higher market share is less persistent, because investment rates are directly influenced by capital stocks of the own firm and of the competitor. The entrant just has to increase its own capital stock in order to reduce investments of the incumbent. A second reason for anticipatory underinvestment by the incumbent is that profits are lower in a Markov perfect equilibrium. This reduces the incentive to invest in this market.

Second, we study a framework where market entry requires incurring a fixed entry cost. This enables the incumbent to establish the critical capital stock level it needs to build up in order to deter entry. Entry deterrence is optimal if it is not too costly to build up this level. We establish that for low entry cost entry accommodation will result, for intermediate levels of the entry cost the incumbent will deter entry, and higher entry cost levels imply that the incumbent is a natural monopoly.

The paper essentially contributes to two streams of the literature. The first stream considers an incumbent-entrant framework where the incumbent has the choice between deterring and accommodating entry. The first contributions are Dixit (1979, 1980), Spence (1977), being surveyed in Tirole (1988, Chap. 8). Maskin (1999) extends this literature by adding uncertainty and obtains that the incumbent should hold a higher capacity to deter the entrant. Abbring and Campbell (2007) construct a discrete time model and find that it may happen that incumbents will serve the total market if entry barriers exist for new entrants. Fudenberg and Tirole (1983, 1986) employ a continuous-time model to find that in a Markov perfect equilibrium it is possible that a firm that has a head start in industry

☆ This research was supported by the Austrian Science Fund (FWF) under Grant P25275-G11.

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http://dx.doi.org/10.1016/j.ejor.2015.04.030

can deter entry (or at least mobility) by overinvestment. In their model the firms have linear investment costs. They further assume that each firm has an upper bound on the amount of investment and argue that this "serves as a proxy for the more realistic case of convex costs of investment" (Fudenberg & Tirole, 1983, p. 230). Our paper in fact considers this convex cost of investment case, which enables us to explicitly consider how firms accumulate capital over time.

The second stream is the literature on duopoly differential games with the emphasis on capital accumulation. Early contributions include Driskill and McCafferty (1989), Reynolds (1987, 1991), with an overview being provided by Dockner, Jø rgensen, Long, and Sorger (2000). Jun and Vives (2004) compare steady states of open-loop and Markov perfect equilibria, where a full characterization is provided in the linear-quadratic case. We do the same, but where Jun and Vives concentrate on a symmetric game, we consider the incumbent-entrant framework.

This paper extends the static literature on entry deterrence to a dynamic framework. This has also been done in Huisman and Kort (in press) but there firms were allowed to invest only once during a time period of infinite length. In our setting firms are allowed to invest whenever they want, resulting in various incremental changes of the capital stock. We find that in an open-loop information structure the incumbent anticipates entry by overinvesting, whereas the incumbent slightly underinvests in a Markov-perfect equilibrium. Furthermore, a policy of entry deterrence is more worthwhile to pursue in the open-loop framework.

The paper is organized as follows. Section 2 presents the model where entry time is fixed and the entrant can enter the market for free. In Section 3 fixed entry costs are added and the entrant may enter from some given time in the future onwards. Section 4 concludes.

2. The model with fixed entry time

Consider an incumbent-entrant model, where, before (eventual) entry takes place, the market consists of one monopolistic firm, being the incumbent (firm 1). The firm that considers entry is denoted by firm 2. This section considers a scenario where the entry time, T, is exogenously given and known, and entry costs are negligible. This implies that we take firm 2's entry at time T for granted, and our aim is to analyze the effect of firm 2's entry on firm 1's investment behavior prior to entry time T. Section 3 deals additionally with positive entry costs and the timing of entry, provided it takes place at all (positive entry costs may result in a profitable policy of entry deterrence by firm 1).

Firm 1's corresponding model builds on the classical capital accumulation models (see, among many others, Barucci, 1998; Eisner & Strotz, 1963). The capital stock, $K_1(t)$, can be increased by capital investments $I_1(t)$, and decreases with a non-negative depreciation rate $\delta > 0$:

$$\dot{K}_1(t) = I_1(t) - \delta K_1(t), \quad K_1(0) = K_{10}, \tag{1}$$

where K_{10} denotes the initial capital stock at $t = 0$. From now on we assume that $K_{10} = 0$.[2] The capital stock $K_1(t)$ is used to produce output with a linear production function, i.e. $F(K_1(t)) = aK_1(t)$ (without loss of generality we chose $a = 1$). The price of output is determined by an inverse demand function, i.e.

$$p(t) = A - K_1(t), \tag{2}$$

with A being a positive constant. Firm 1's revenue therefore equals

$$R_1(t) = p(t)K_1(t) = (A - K_1(t))K_1(t). \tag{3}$$

The cost of investment consists of linear acquisition costs, $bI_1(t)$, and quadratic costs of implementation, $\frac{c}{2}I_1^2(t)$, where b and c are positive constants.

At entry time T the market switches to competition. The present value of the incumbent's profits earned from there on depends on its capital stock at the switching time $K_1(T)$, and, since the initial capital stock of the entrant equals zero,[3] we can denote these profits by $S(K_1(T))$. This results in the following optimization problem for the incumbent:

$$\max_{I_1(t)} \int_0^T e^{-rt}\left((A - K_1(t))K_1(t) - bI_1(t) - \frac{c}{2}I_1^2(t)\right)dt + e^{-rT}S(K_1(T))$$
$$\text{s.t. } \dot{K}_1(t) = I_1(t) - \delta K_1(t), \quad K_1(0) = 0, \tag{4}$$

where r is the discount rate.

From time T on, two firms compete in an oligopolistic market with homogenous goods. Consequently, the output price after firm 2 has entered, equals

$$p(t) = A - K_1(t) - K_2(t) \tag{5}$$

for both firms. The firms are both profit maximizers, where the time horizon is infinite. Putting things together we arrive at a classical capital accumulation game as presented in Reynolds (1987),[4] i.e.

$$\begin{aligned}
&\text{Firm } 1: \max_{I_1(t)} \\
&\qquad \int_T^\infty e^{-rt}\left((A - K_1(t) - K_2(t))K_1(t) - bI_1(t) - \frac{c}{2}I_1^2(t)\right) dt,\\
&\text{Firm } 2: \max_{I_2(t)} \\
&\qquad \int_T^\infty e^{-rt}\left((A - K_1(t) - K_2(t))K_2(t) - bI_2(t) - \frac{c}{2}I_2^2(t)\right) dt,\\
&\text{s.t.} \quad \dot{K}_1(t) = I_1(t) - \delta K_1(t),\\
&\qquad \dot{K}_2(t) = I_2(t) - \delta K_2(t), \quad K_2(T) = 0.
\end{aligned} \tag{6}$$

In the same paper this differential game is solved for the open-loop and feedback (or Markov perfect) case. Therefore, we will not repeat the analysis, but only some highlights and key results we need for our economic analysis (see Appendices A–C). Due to the linear quadratic structure it is possible to obtain an analytical solution. This is presented in the following sections for both (open-loop and Markov perfect equilibrium) scenarios.

2.1. Analysis and economic interpretation (Markov perfect equilibrium case)

In the remainder of the paper the superscripts M, MP, and O denote variables that correspond to monopoly (M), or Markov perfect (MP) and open-loop commitment structure (O).

This section deals with a Markov perfect equilibrium structure in the duopoly game that arises after firm 2 has entered. As demonstrated in Reynolds (1987), the Hamilton–Jacobi–Bellman function has 6 solutions, where one is asymptotically stable (for details we refer to Reynolds (1987)). Comparing the steady state solutions reveals that the capital stock as well as the investments of the monopolist always exceed that of a duopoly firm (Markov perfect equilibrium), i.e.

$$\hat{K}_1^M = \frac{A - b(r+\delta)}{2 + \delta c(r+\delta)} > \frac{A - b(r+\delta)}{3 + \delta c(r+\delta) - \frac{\sigma}{\pi - c(r+\delta)}} = \hat{K}_1^{\mathrm{MP}},$$

[2] The analysis of the model with positive K_{10} is completely analogous. Note that this choice is no restriction to the model. Due to the Bellman principle the behavior of the incumbent before time T with positive K_{10} (i.e. $K_{10} = \xi > 0$, entry time T) is completely the same compared to the situation in which the incumbent owns that capital stock at some time $\bar{t}$ with entry time horizon $T + \bar{t}$.

[3] Note that the analysis of this paper is also possible for a positive initial capital stock of the competitor. However, it is more reasonable to assume within this model that the capital stock has to be built up after the entrance.

[4] For other contributions we refer to Dragone, Lambertini, and Palestini (2010), Fershtman and Muller (1984), Reynolds (1991). A capital accumulation game with a capital stock with vintage structure has been dealt with in Wrzaczek and Kort (2012). For a profound overview on dynamic capital accumulation games we refer to Dockner et al. (2000), Long (2010).