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Surrogate upper bound sets for bi-objective bi-dimensional binary knapsack problems

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ABSTRACT

The paper deals with the definition and the computation of surrogate upper bound sets for the bi-objective bi-dimensional binary knapsack problem. It introduces the Optimal Convex Surrogate Upper Bound set, which is the tightest possible definition based on the convex relaxation of the surrogate relaxation. Two exact algorithms are proposed: an enumerative algorithm and its improved version. This second algorithm results from an accurate analysis of the surrogate multipliers and the dominance relations between bound sets. Based on the improved exact algorithm, an approximated version is derived. The proposed algorithms are benchmarked using a dataset composed of three groups of numerical instances. The performances are assessed thanks to a comparative analysis where exact algorithms are compared between them, the approximated algorithm is confronted to an algorithm introduced in a recent research work.
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1. Introduction

1.1. Problem formulation and characteristics

Given a set of items $J = \{1, \ldots, n\}$ and a set of dimensions $I = \{1, \ldots, m\}$, we associate with each dimension $i \in I$ a capacity $\omega_i \in \mathbb{N}_1$ and a set of weights $w_{ij} \in \mathbb{N}_1$ for each item $j \in J$. Moreover, each item is available in a single copy. The *multi-objective multi-dimensional binary knapsack problem* consists in packing a subset of J into a container with limited capacity over the dimensions I. This must be done while maximizing a profit according to a set of objectives $K = \{1, \ldots, p\}$. For this, a profit $c_j^k \in \mathbb{N}_1$ is associated with each item $j \in J$ according to each objective $k \in K$. The general formulation of this problem is the following

$$\begin{aligned} \max \sum_{j=1}^{n} c_j^k x_j & \qquad k = 1, \ldots, p \\ \text{s.t.} \sum_{j=1}^{n} w_{ij} x_j \le \omega_i & \qquad i = 1, \ldots, m \\ x_j \in \{0, 1\} & \qquad j = 1, \ldots, n \end{aligned} \qquad (pOmDKP)$$

This problem has received a lot of attention, see *e.g.* Fréville (2004), Varnamkhasti (2012), Lust and Teghem (2012) for recent surveys. It is used for representing various practical problems as capital budgeting, allocating processors (Kellerer, Pferschy, & Pisinger, 2004; Martello & Toth, 1990; da Silva, Clímaco, & Figueira, 2004). For example, Clausen, Hjorth, Nielsen, and Pisinger (2010) consider a special case of two-dimensional knapsack for the problem of assigning seats in a train for a group of people traveling together.

Many particular cases of this problem have been studied, and all of them are $\mathcal{NP}$-hard. We will denote this problem by (*pOmDKP*), according to the number of objectives and dimensions. We will omit the number of dimensions and/or the number of objectives in this notation whenever $p = 1$ and/or $m = 1$. In the following, we will make a particular distinction between the single-dimensional ($m = 1$) and the multi-dimensional ($m > 1$) case. Indeed, (*KP*) problem can be considered as an "easy" $\mathcal{NP}$-hard problem as several practically efficient methods have been proposed for its exact solution (Kellerer et al., 2004; Martello, Pisinger, & Toth, 1999; Pisinger, 1994). Their theoretical time-complexity is pseudo-polynomial. (2*OKP*) problem is also one of the most studied multi-objective combinatorial optimization problems. A number of methods have been proposed for its exact solution (Bazgan, Hugot, & Vanderpooten, 2009; Delort & Spanjaard, 2010; Jorge, 2010; Ulungu & Teghem, 1997; Visée, Teghem, Pirlot, & Ulungu, 1998). Bazgan et al. (2009); Jorge (2010) have also provided exact solution methods for (3*OKP*) problem.

(*mDKP*) problem is practically far more difficult than (*KP*) problem as pointed out by Fréville (2004). In the multi-objective case,

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instances of this problem are often used as a benchmark to compare metaheuristics (Ishibuchi, Hitotsuyanagi, Tsukamoto, & Nojima, 2009; Jaszkiewicz, 2004; Tricoire, 2012; Zitzler & Thiele, 1999). However, this problem has received less attention in an exact context (Florios, Mavrotas, & Diakoulaki, 2010; Lust & Teghem, 2012; Mavrotas, Figueira, & Antoniadis, 2011). To our knowledge, only one exact method has been proposed for the specific (2O2DKP) case (Gandibleux & Perederieieva, 2011; Perederieiva, 2011).

In this paper, we also consider the (2O2DKP) problem. The main purpose is to design and compute a tight upper bound set for this problem. Such a bound set is a crucial component when it is embedded within an algorithm aiming to enumerate a complete set of nondominated points of the (2O2DKP).

1.2. Efficiency, nondominance and bound sets

The main definitions, properties, and notations of multi-objective combinatorial optimization are given now. A more complete introduction can be found in Ehrgott (2005).

A multi-objective combinatorial optimization problem can be formulated as

$$\max\{(z_1(x), \dots, z_p(x)) = Cx : x \in X\}, \qquad (MOCO)$$

with a linear objective matrix $C \in \mathbb{R}^{p \times n}$, variables $x \in \mathbb{R}^n$, and the feasible set (in decision space $\mathbb{R}^n$)

$$X := \{x \in \{0, 1\}^n : Ax \leqq b,\ x \geqq 0\}.$$

Matrix A is an $m \times n$ matrix of constraints and $b \in \mathbb{R}^m$ the right hand side vector. The image of X under C, *i.e.*,

$$Y := CX := \{y = Cx \in \mathbb{R}^p : x \in X\}$$

is called the outcome set in objective space $\mathbb{R}^p$.

We assume that no feasible solution optimizes all objectives simultaneously and use the following notations for componentwise orders in $\mathbb{R}^p$. Let $y^1, y^2 \in \mathbb{R}^p$. We write $y^1 \geqq y^2$ (y^1 *weakly dominates* y^2) if $y^1_k \geq y^2_k$ for $k = 1, \dots, p$; $y^1 \geq y^2$ (y^1 *dominates* y^2) if $y^1 \geqq y^2$ and $y^1 \neq y^2$; and $y^1 > y^2$ (y^1 *strictly dominates* y^2) if $y^1_k > y^2_k$, $k = 1, \dots, p$. We define $\mathbb{R}^p_{\geqq} := \{x \in \mathbb{R}^p : x \geqq 0\}$ and analogously $\mathbb{R}^p_{\geq}$ and $\mathbb{R}^p_{>}$.

A feasible solution $\hat{x} \in X$ is called *efficient (weakly efficient)* if there does not exist $x \in X$ such that $z(x) \geq z(\hat{x})$ ($z(x) > z(\hat{x})$). If $\hat{x}$ is (weakly) efficient, then $z(\hat{x})$ is called *(weakly) nondominated*. The *efficient set* $X_E \subseteq X$ is defined as

$$X_E := \{x \in X : \nexists\, \bar{x} \in X : z(\bar{x}) \geq z(x)\},$$

and its image in objective space is referred to as the *nondominated set* $Y_N := z(X_E)$. Equivalently, Y_N can be defined by $Y_N := \{y \in Y : (y + \mathbb{R}^p_{\geq}) \cap Y = \{y\}\}$. This concept is extended by defining $S_N := \{s \in S : (s + \mathbb{R}^p_{\geq}) \cap S = \{s\}\}$ for an arbitrary set $S \in \mathbb{R}^p$. The exact solution of a multi-objective combinatorial optimization problem consists in determining a *complete set* of efficient solutions, *i.e.* to determine at least one efficient solution for each nondominated point.

As we consider here multi-objective combinatorial optimization problems, several classes of efficient solutions need to be distinguished. *Supported* efficient solutions are optimal solutions of a weighted sum single objective problem (Geoffrion, 1968)

$$\max\{\lambda_1 z_1(x) + \cdots + \lambda_p z_p(x) : x \in X\} \qquad (MOCO_\lambda)$$

for some $\lambda \in \mathbb{R}^p_{>}$. Their images in objective space are supported nondominated points. We use the notations X_{SE} and Y_{SN}, respectively. In order to avoid a confusion with the weights of the items of (*pOmDKP*), the weight vector $\lambda \in \mathbb{R}^p_{>}$ will be called *direction* in the following. All supported nondominated points are located on the boundary of the convex hull of Y (convY), *i.e.*, they are nondominated points of (convY) $- \mathbb{R}^p_{\geqq}$.

Nonsupported efficient solutions are efficient solutions that are not optimal solutions of ($MOCO_\lambda$) for any direction $\lambda \in \mathbb{R}^p_{>}$. Nonsupported nondominated points are located in the interior of the convex hull of Y. In the particular bi-objective case, nonsupported nondominated points are located in the interior of triangles the hypotenuse of which is defined by consecutive supported nondominated points with respect to z_1. The set of nonsupported efficient solutions and nondominated points are denoted respectively by X_{NE} and Y_{NN}.

Finally, we can distinguish two classes of supported efficient solutions. The set of *extremal* supported efficient solutions X_{SE1} is a subset of X_{SE} the corresponding point of which is an extreme point of convY. $Y_{SN1} := z(X_{SE1})$ is the set of *nondominated extreme points*. $X_{SE2} := X_{SE} \setminus X_{SE1}$ and $Y_{SN2} := Y_{SN} \setminus Y_{SN1}$ are respectively the sets of *nonextremal* supported efficient solutions and nondominated points. The computation of the set Y_{SN1} can be easily done in the bi-objective context using the algorithm by Aneja and Nair (1979), under the assumption that the corresponding single-objective problem can be solved efficiently in practice.

Bounds on the optimal value of a single-objective problem are crucial to design efficient solution methods in the single-objective case. Their generalization to the multi-objective case called *bound sets* (Ehrgott & Gandibleux, 2001) are used to bound Y_N. Several definitions of bound sets have been proposed, we use the definition proposed by Ehrgott and Gandibleux (2007). Contrary to the single-objective case, the definitions of upper and lower bound sets are not symmetric. A lower bound set for Y_N is generally given by a set of known feasible points filtered by dominance. Upper bound sets for Y_N (or for any subset of points in $\mathbb{N}^p$) can be obtained by far more various ways.

We introduce some additional terminology before the next definition. S is $\mathbb{R}^p_{\geqq}$*-closed* if the set $S - \mathbb{R}^p_{\geqq}$ is closed and $\mathbb{R}^p_{\geqq}$*-bounded* if there exists $s^0 \in \mathbb{R}^p$ such that $S \subset s^0 - \mathbb{R}^p_{\geqq}$.

Definition 1 (Ehrgott & Gandibleux, 2007)**.** Let $\bar{Y} \subset \mathbb{R}^p$. An upper bound set U for $\bar{Y}$ is an $\mathbb{R}^p_{\geqq}$-closed and $\mathbb{R}^p_{\geqq}$-bounded set $U \subset \mathbb{R}^p$ such that $\bar{Y} \subset U - \mathbb{R}^p_{\geqq}$ and $U \subset (U - \mathbb{R}^p_{\geq})_N$.

In solution methods, $\bar{Y}$ denote generally the set of nondominated points of a subproblem of the considered problem (*e.g.* a relaxation). The same way as the single-objective context, bound sets for $\bar{Y}$ must be as tight as possible for a reasonable computational cost. Ehrgott and Gandibleux (2007) have proposed the notion of dominance between bound sets.

Definition 2 (Ehrgott & Gandibleux, 2007)**.** Given two upper bound sets U_1 and U_2 for a same set $\bar{Y}$, U_1 dominates U_2 if $U_1 \subset U_2 - \mathbb{R}^p_{\geqq}$ and $U_1 - \mathbb{R}^p_{\geqq} \neq U_2 - \mathbb{R}^p_{\geqq}$.

The dominance relation between bound sets is transitive. Nevertheless, if we can always compare bound values in the single-objective case, dominance between bound sets does not necessarily occur (Fig. 1). Computing several upper bound values in the single-objective context, the smallest defines naturally the tightest bound. Proposition 1 provides a way to merge upper bound sets.

Proposition 1 (Ehrgott & Gandibleux, 2007)**.** *If U_1 and U_2 are upper bound sets for a same set $\bar{Y}$ and $U_1 - \mathbb{R}^p_{\geqq} \neq U_2 - \mathbb{R}^p_{\geqq}$ then $U^* := [(U_1 - \mathbb{R}^p_{\geq}) \cap (U_2 - \mathbb{R}^p_{\geqq})]_N$ is an upper bound set for $\bar{Y}$ dominating U_1 and U_2.*

It is interesting to note that the simultaneous use of several upper bound sets can thus be a way to obtain a particularly tight upper bound set (Fig. 2). We provide some additional explanations based on Figs. 1 and 2. We consider here polyhedral upper bound sets U defined by a set of extreme points $\{y^1, \dots, y^k\}$, *i.e.* $U = (\text{conv}\{y^1, \dots, y^k\} - \mathbb{R}^2_{\geqq})_N$. $U_1 = (\text{conv}\,\{y_1, y_2, y_3\})_N$ and $U_2 = (\text{conv}\,\{y_4, y_5, y_6\})_N$ are incomparable bound sets, i.e. there is no dominance relation between them. The bound set $U^* := [(U_1 - \mathbb{R}^2_{\geqq}) \cap (U_2 - \mathbb{R}^2_{\geqq})]_N$ (which

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