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Decision Support

# Generic constraints handling techniques in constrained multi-criteria optimization and its application

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ABSTRACT

This paper investigates the *constraints handling technique* (CHT) in algorithms of the *constrained multi-criteria optimization problem* (CMOP). The CHT is an important research topic in constrained *multi-criteria optimization* (MO). In this paper, two simple and practicable CHTs are proposed, where one is a nonequivalent relaxation approach which is much suitable for the constrained *multi-criteria discrete optimization problem* (MDOP), and the other is an equivalent relaxation approach for the general CMOP. By using these CHTs, a CMOP (i.e., the *primal problem*) can be transformed into an unconstrained *multi-criteria optimization problem* (MOP) (i.e., the *relaxation problem*). Based on the first CHT, it is theoretically proven that the efficient set of the primal CMOP is a subset of the strictly efficient set $\overline{\mathbb{E}}$ of the relaxation problem and can be extracted from $\overline{\mathbb{E}}$ by simply checking the dominance relation between the solutions in $\overline{\mathbb{E}}$. Follows from these theoretical results, a three-phase based idea is given to effectively utilize the existing algorithms for the unconstrained MDOP to solve the constrained MDOP. In the second CHT, the primal CMOP is equivalently transformed into an unconstrained MOP by a special relaxation approach. Based on such a CHT, it is proven that the primal problem and its relaxation problem have the same efficient set and, therefore, general CMOPs can be solved by utilizing any of the existing algorithms for the unconstrained MOPs. The implementing idea, say *two-phase based idea*, of the second CHT is illustrated by implanting a known MOEA. Finally, the two-phase based idea is applied to some of the early MOEAs and the application performances are comprehensively tested with some benchmarks of the CMOP.



## 1. Introduction

A *multi-criteria optimization* (MO) *problem* (MOP) has a rather different perspective compared with one having a single objective. To rank solutions of MOPs, some comparing criteria such as the dominance relation and the strict dominance relation between solutions are defined. The set of solutions which are not dominated by any solution in the solution space is referred to the *Pareto optimal set* (POS) or *efficient set* (ES). Similarly, a set containing solutions which are not strictly dominated by any solution in the solution space is referred to the *strict Pareto optimal set* (SPOS) or *strictly efficient set* (SES). Generally, in a single objective optimization context, there is only one global optimum objective value, but in a MOP there is a set of solutions, i.e., the ES or SES, which are considered to be equally important; all of them constitute the global optimum solutions.

The MOPs are usually classified into two categories, say *constrained MOP* (CMOP) and *unconstrained MOP* (UCMOP), depending on whether the MOP has constraints or not. The *constraints handling technique* (CHT) in the solution algorithms of CMOPs is one of the major research topics and, consequently, lots of approaches with respect to the CHT are developed. According to the characteristics of different constraints handling approaches, Coello Coello, Veldhuizen, and Lamont (2002) grouped them into five types. Some of them will be reviewed in Section 2. This paper will focus on proposing some new CHTs.

We know that the original Lagrangean relaxation approach is a constraints handling technique for the single objective constrained optimization problems and has been extended into CMOPs. In this paper, we propose two relaxation based CHTs in sense of equivalent and nonequivalent, respectively, where *equivalent* implies that the primal CMOP and its relaxation problem have the same optimal solutions, i.e., the same efficient set. The essential of these two CHTs is to convert a CMOP (the *primal problem*) into an UCMOP (*relaxation problem*) by simply appending one or more special objectives associated with the constraints to the objective vector of the CMOP and simultaneously removing its constraints.

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The first of the proposed CHTs provides a nonequivalent relaxation approach and, based on this CHT, it is proven that the efficient set of the primal CMOP is a subset of the strictly efficient set $\overline{\mathbb{E}}$ of the corresponding relaxation problem and can be extracted from $\overline{\mathbb{E}}$ by simply checking the dominance relation between the solutions in $\overline{\mathbb{E}}$. From these theoretical results, a three-phase based idea is given to utilize any of the known algorithms for the unconstrained MOP to solve the MOPs. Because we must find the strictly efficient set of the relaxation problem, a defect coming with such a three-phase based CHT is that the size of search space will be increased. In addition, the relaxation approach in this CHT gives rise to that the linear properties of the primal CMOP might be changed. So this CHT is further improved such that the linear properties of the primal CMOP can be maintained. Note that, in an algorithm for a *combinatorial optimization problem* (COP), it is usually very difficult to deal with the nonlinear constraints or objectives. Therefore, maintaining such linear properties in the relaxed problem is very important for the related algorithm when handle COPs' constraints.

The second of the proposed CHTs provides an equivalent relaxation approach by simply redefining all of the objectives of the CMOP and appending only one specially defined objective to the objective vector of the primal CMOP. By such a way, the primal CMOP is thus transformed into an unconstrained MOP. Based on such a CHT, it is proven that the primal problem and relaxation problem have the same efficient set and a two-phase based CHT is proposed for the general CMOP. Though both of the three-phase based and two-phase based CHTs can be applied to any CMOP, the former is very convenient for the constrained *multi-criteria discrete optimization problem* (MDOP) while the latter is very suitable for the general CMOP.

Over the past decades, *evolutionary algorithms* (EAs) have been applied to various fields and a number of *multi-criteria optimization evolutionary algorithms* (MOEAs) were proposed. A full review of recent and early approaches with respect to MOEAs can be found in Zhou, Qu, Li, Zhao, and Suganthan (2011) and Deb (2001), respectively. The main reason for the popularity of EAs for solving MOPs is their population-based nature and ability to find multiple optima simultaneously (Coello Coello et al., 2002). Some early important EAs are the *strength Pareto evolutionary algorithm* (SPEA) of Zitzler and Thiele (1998, 1999), *Pareto archived evolution strategy* (PAES) of Knowles and Corne (1999), *nondominated sorting* genetic algorithm *(GA)* (NSGA) of Srinivas and Deb (1995) and *indicator-based evolutionary algorithm* (IBEA) of Zitzler and Kunzli (2004). Thereafter, the SPEA, PAES and NSGA were improved by Zitzler, Laumanns, and Thiele (2001), Knowles and Corne (2000) and Deb, Pratap, Agarwal, and Meyarivan (2002), respectively, and the improved versions are identified as SPEA-II, PAES-II and NSGA-II, respectively. The SPEA-II, PAES-II and NSGA-II are three prominent MOEAs used when comparing a newly designed MOEA (Coello Coello et al., 2002). Furthermore, a later MOEA with good performance is the archive based *simulated annealing* (SA), say AMOSA, of Bandyopadhyay, Saha, Maulik, and Deb (2008). By extending the idea of flexible integration of preference information of Fonseca and Fleming (1998b) and Knowles (2002), Zitzler and Kunzli (2004) proposed a general IBEA for the general unconstrained MOP. The main idea of IBEA is to formalize preferences in terms of continuous generalizations of the dominance relation, which leads to a simple algorithmic concept. Thereby, IBEA not only allows adaptation to arbitrary preference information and optimization scenarios, but also does not need any diversity preservation techniques, in contrast to Fonseca and Fleming (1998b). In comparison to Knowles (2002), IBEA is more general, since the population size can be arbitrary, and faster, because it only compares pairs of individuals and not entire approximation sets. These MOEAs will be employed in testing the proposed two-phase based CHT in this paper.

From the view point of application, the proposed CHTs have high adaptability or flexibility and their applications are extremely simple and convenient. In order to apply them to solve a CMOP, we only need to define a new UCMOP by utilizing the proposed CHTs and then solve the new problem with any of the existing solution algorithms of the UCMOP. In other words, if exists an algorithm for the UCMOP, then this algorithm can also be applied to solve the CMOP by combing it with the proposed CHTs. In this paper, the main idea of the proposed two-phase based CHT is illustrated by combining it with the AMOSA and NSGA-II, respectively.

This paper is organized as follows. Firstly, some of the research literatures with respect to CMOP are surveyed in Section 2 and notations regarding multi-criteria optimization are introduced in Section 3. Secondly, the nonequivalent and equivalent relaxation approaches are proposed and some theoretical results are proven in Sections 4.1 and 4.2, respectively. Additionally, a handling technique for the strict constraints is introduced in Section 4.3. Based on the theoretical results in Sections 4.1 and 4.2, a three-phase based idea for the CHT of the constrained MDOPs and a two-phase based idea for the CHT of the CMOPs are introduced in Section 4.4, and the two-phase based idea is illustrated by utilizing a known MOEA, i.e., NSGA-II. Finally, some metrics to assess the performances of MOEAs are given in Section 5 and the proposed two-phase based idea and its application effect are tested in Section 6 via some known CMOP test examples.

## 2. Literature review

Many constraint handling methods have been proposed to solve constrained *scalar objective optimization problems* (SOPs) (e.g., Michalewicz & Schoenauer, 1996). According to the characteristics of different constraint handling methods, Coello Coello (2002) grouped them into five categories: (1) penalty functions; (2) special representations and operators; (3) repair algorithms; (4) separate objective and constraints; and (5) hybrid methods. Although not all constraint handling approaches developed for scalar objective optimization are suitable for CMOPs, some of them have been successfully extended into the constrained multiobjective areas (Coello Coello et al., 2002). This section will introduce some of the constraint handling approaches used for constrained multi-criteria optimization in last decade. Because the evolutionary strategy based approach is the most possible and effective solution approach for generic MOPs at present, the existing CHTs for CMOPs are that of some special handling approaches together with the evolutionary operations of EAs. Some of the evolutionary strategy based CHTs will be reviewed in the remainder of this section.

The penalty function approach is known as the most popular and oldest constraint handling method. It was firstly introduced by Courant (1943) and then extended to CMOPs by many researchers (e.g., Das, Natarajan, Stevens, & Koduru, 2008; Woldesenbet, Yen, & Tessema, 2009). Its basic idea is to transform a CMOP into an unconstrained one by introducing a penalty term into the primal fitness function to penalize constraint violations (Cai & Wang, 2006). There have been several schemes to impose suitable penalties when solve CMOPs, including the death penalty, static penalty, dynamic penalty, and adaptive penalty (Coello Coello, 2002). Woldesenbet et al. (2009) introduced a *very promising self-adaptive penalty function* for the constrained multi-criteria evolutionary optimization. This method tracks the percentage of the feasible solutions in the current population to determine the amount of penalty to be added. A small percentage of feasible individuals results in a larger penalty while a larger percentage generates a small penalty factor. This technique is able to balance information extraction from feasible and infeasible solutions (Zhou et al., 2011).

The immune strategy has also been commonly used to handle constraints. Zhang (2007) proposed a constrained nonlinear multi-criteria optimization immune algorithm based on the humoral immune response principle and ideas of T-cell regulation. This algorithm adopts and modifies a uniform designing scheme to provide an alternative feasible solutions set for dealing with