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Decision Support

Generalized analytic network process

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ABSTRACT

The analytic network process (ANP) is a methodology for multi-criteria decision making used to derive priorities of the compared elements in a network hierarchy, where the dependences and feedback within and between the elements can be considered. However, the ANP is limited to the input preferences as crisp judgments, which is often unfavorable in practical applications. As an extension of the ANP, a generalized analytic network process (G-ANP) is developed to allow multiple forms of preferences, such as crisp (fuzzy) judgments, interval (interval fuzzy) judgments, hesitant (hesitant fuzzy) judgments and stochastic (stochastic fuzzy) judgments. In the G-ANP, a concept of complex comparison matrices (CCMs) is developed to collect decision makers' preferences in the multiple forms. From a stochastic point of view, we develop an eigenvector method based stochastic preference method (EVM-SPM) to derive priorities from CCMs. The main steps of the G-ANP are summarized, and the implementation of the G-ANP in Matlab and Excel environments are given in detail, which is also a prototype for a decision support system. A real-life example of the piracy risk assessment to the energy channels of China is proposed to demonstrate the G-ANP.

1. Introduction

In multi-criteria decision making, the analytic hierarchy process (AHP) (Saaty, 1980) is a popular methodology to derive priorities of compared elements (or objectives, alternatives etc.) to assist decision makers (DMs) to make decisions. It has been successfully applied in practice (Saaty, 1989, 2008). In the AHP, the DMs provide their preferences over paired comparisons of elements by a 1–9 scale in each level of a hierarchy. The hierarchy of the AHP is a linear top down form with clear independent levels, where the elements in each level are also independent.

In the AHP, prioritization methods are necessary to derive priorities of the compared elements in each level of the hierarchy, such as the eigenvector method (EVM) (Saaty, 1977), the logarithmic least squares method (LLSM) (Crawford & Williams, 1985), and the logarithmic goal programming method (GPM) (Bryson, 1995). Due to the inherent difference of the DMs, their preferences can be inconsistent. As discussed by Saaty and Vargas (1987), only the EVM takes consistency of the preferences into account. The use of other prioritization methods may lead to wrong decisions by reversing ranks. Therefore, consistency checking and improving are significant in the AHP to guarantee meaningful results.

The AHP has a limitation that it cannot deal with interactions and dependencies between the elements in the levels of the hierarchy. For example, to predict the market share of cell-phone providers, the elements that influence the market share of a company can be costs and services, where the services may also influence the costs. To overcome this limitation, Saaty (2001) considered the dependences and feedback of the elements, and then developed the analytic network process (ANP).

In the ANP, the network allows clusters of elements influence each other, or has loops if the elements in the clusters have inner dependences. So the network spreads out in all directions and its cluster of elements are not arranged in a particular order (Saaty, 2004). The advantage of the ANP in dealing with dependences and feedback enables it to be very useful in many practical applications. For example, the ANP has been used for the interdependent information system project selection (Lee & Kim, 2000), the R&D project selection (Meade & Presley, 2002), the logistics service provider selection (Jharkharia & Shankar, 2007), the product mix planning (Chung, Lee, & Pearn, 2005), the SWOT analysis (Yüksel & Dagdeviren, 2007), the financial-crisis forecasting (Niemira & Saaty, 2004), and the multi-criteria analysis (Wolfslehner, Vacik, & Lexer, 2005).

However, the preferences in the ANP are limited to crisp judgments based on the 1–9 scale. In many cases, the DMs may prefer to many other possible forms to represent their preferences. To overcome this limitation of the ANP, Mikhailov and Singh (2003) developed the fuzzy analytic network process (F-ANP) which allows

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multiple forms of preferences, such as crisp judgments, interval judgments, and fuzzy judgments. Moreover, the main difference between the F-ANP and the ANP is that the F-ANP uses a fuzzy preference programming (FPP) method as an alternative prioritization method to the EVM.

The FPP is a linear programming method that maximizes the DMs' satisfaction degree to consistency of the preferences, where a consistency index is also provided. However, this consistency index is a relative index varying with a so called "deviation parameter", which cannot be used to measure whether the preferences are consistent or not. Therefore, similar to the LLSM and the GPM, the FPP also does not take consistency of the preferences into account. Although the FPP is convenient to be used to derive priorities, it may result in ineffective decisions in some cases if consistency of the preferences cannot be guaranteed.

As complexity of the socio-economic environment is increased, more uncertainties are experienced by the DMs apart from crisp judgments, interval judgments and fuzzy judgments. For example, the DMs may be hesitant about several possible values for a judgment. This concept of hesitance of being used for describing the preferences in decision making is based on hesitant fuzzy sets originally introduced by Torra (2010). Based on a 0.1–0.9 scale, the judgments that each is characterized by several possible values are called hesitant fuzzy judgments. They have been widely used in decision making problems (Xia & Xu, 2011; Xia, Xu, & Chen, 2013; Xu & Xia, 2011). Based on the 1–9 scale, hesitant judgments were also proposed for decision making (Xia & Xu, 2013; Zhu & Xu, 2014a).

The judgments can be also indicated by stochastic variables, which have been considered by some researchers as stochastic judgments. For example, Rosenbloom (1997) claimed that the subjective preferences with a continuous probability distribution is a standard requirement in the conventional decision analysis, so stochastic judgments should be used rather than crisp judgments and interval judgments. Moskowitz, Tang, and Lam (2000) recommended to use stochastic variables to account for inconsistency and imprecision in the preferences. Hahn (2003) claimed that the deterministic methods in decision making are special cases of their stochastic counterparts, and then proposed a stochastic formulation of the AHP with stochastic judgments.

Based on the discussion above, so many possible forms of preferences are available for the DMs. In practice, different forms of preferences shall be useful in different situations. However, different forms of preferences also require different prioritization methods. Is there a general method can deal with all the possible forms of preferences mentioned above and produce meaningful results? In this paper, we develop a generalized analytic network process (G-ANP) method as an extension of the ANP. In the G-ANP, the DMs can provide their preferences over paired comparisons of elements in the network as crisp (fuzzy) judgments, interval (interval fuzzy) judgments, hesitant (hesitant fuzzy) judgments and stochastic (stochastic fuzzy) judgments, etc.

Since the preferences in the G-ANP can be in multiple forms, we define them as complex judgments to construct complex comparison matrices (CCMs). An expected index is developed to measure whether or not a CCM is of the acceptable consistency. Then an automatic consistency improving method is proposed to repair inconsistent CCMs to guarantee meaningful results. Consistency checking and improving of the G-ANP also eliminate the drawbacks of the F-ANP that cannot take consistency into account. To derive priorities from CCMs, we develop an eigenvector method based stochastic preference method (EVM-SPM) as a new prioritization method. Based on these new developed methods, a step by step procedure of the G-ANP is proposed.

The rest of the paper is organized as follows. Section 2 introduces all the possible forms of preferences in the G-ANP, and then defines complex judgments and CCMs. Section 3 focuses on consistency checking and improving of CCMs. Section 4 develops the EVM-SPM. In Section 5, the step by step implementation of the G-ANP is discussed in detail. Then a real-life example of the piracy risk assessment to the energy channels of China is given to illustrate the G-ANP in Section 6. Section 7 gives some concluding remarks.

Table 1
The 1–9 scale.

Scale	Meaning
1	Equally preferred
3	Moderately preferred
5	Strongly preferred
7	Very strongly preferred
9	Extremely preferred
Other values between 1 and 9	Intermediate values used to represent compromise

2. Complex judgments and complex comparison matrices

In this section, we introduce the judgments based on a 1–9 scale and a 0.1–0.9 scale respectively, then define complex judgments and complex comparison matrices (CCMs).

2.1. Judgments based on the 1–9 scale

Based on the 1–9 scale (Saaty, 1980), shown in Table 1, the DMs' preferences can be clearly represented by crisp values to describe the relationships between paired comparisons of elements, where the preferences are called crisp judgments.

A reciprocal comparison matrix consisting of crisp judgments, $A = (a_{ij})_{n\times n}$, can be shown as follows:

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \dots & a_{1n} \\ & 1 & a_{23} & \dots & a_{2n} \\ & \vdots & 1 & & \vdots \\ \vdots & 1/a_{ij} & \vdots & \ddots & \vdots \\ & \dots & & & 1 \end{bmatrix} \tag{1}$$

where the judgments satisfy the reciprocal property as $a_{ij} = 1/a_{ji}$, $i, j = 1, 2, \dots, n$.

If a judgment is represented by an interval to indicate uncertainties, then it is called an interval judgment. An interval reciprocal comparison matrix (Saaty & Vargas, 1987) constructed by interval judgments, $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, is given as follows:

$$\tilde{A} = \begin{bmatrix} 1 & [a^l_{12}, a^u_{12}] & [a^l_{13}, a^u_{13}] & \dots & [a^l_{1n}, a^u_{1n}] \\ & 1 & [a^l_{23}, a^u_{23}] & \dots & [a^l_{2n}, a^u_{2n}] \\ & \vdots & 1 & & \vdots \\ \vdots & [1/a^u_{ij}, 1/a^l_{ij}] & \vdots & \ddots & \vdots \\ & \dots & & & 1 \end{bmatrix} \tag{2}$$

where the judgments satisfy the reciprocal property as $a^l_{ij} = 1/a^u_{ji}$, $a^u_{ij} = 1/a^l_{ji}$ and $a^l_{ij} \le a^u_{ij}$, $i, j = 1, 2, \dots, n$.

In accordance with Saaty and Vargas (1987) of considering discrete judgments within the interval, we also use the integers and reciprocals of the integers between the upper and lower bounds of an interval. For example, the interval judgment [1/4,2] has 1/4, 1/3 and 1/2 as reciprocals of integers, and 1 and 2 as integers, for a total of five judgments. So the reciprocal comparison matrix is a special case of the interval reciprocal comparison matrix.

If a judgment includes several possible values, then it is called a hesitant judgment, which can be represented by a set $h = \{a^{(l)} | l = 1, \dots, |h|\}$. For example, a hesitant judgment $\{1/3, 1\}$ has two possible values as 1/3 and 1. A hesitant reciprocal comparison matrix consisting of hesitant judgments, denoted by $H = (h_{ij})_{n\times n}$ with