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Discrete Optimization

A matheuristic approach for the Pollution-Routing Problem

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ABSTRACT

This paper deals with the Pollution-Routing Problem (PRP), a Vehicle Routing Problem (VRP) with environmental considerations, recently introduced in the literature by Bektaş and Laporte (2011) [Transportation Research Part B: Methodological 45 (8), 1232–1250]. The objective is to minimize operational and environmental costs while respecting capacity constraints and service time windows. Costs are based on driver wages and fuel consumption, which depends on many factors, such as travel distance and vehicle load. The vehicle speeds are considered as decision variables. They complement routing decisions, impacting the total cost, the travel time between locations, and thus the set of feasible routes. We propose a method which combines a local search-based metaheuristic with an integer programming approach over a set covering formulation and a recursive speed-optimization algorithm. This hybridization enables to integrate more tightly route and speed decisions. Moreover, two other "green" VRP variants, the Fuel Consumption VRP (FCVRP) and the Energy Minimizing VRP (EMVRP), are addressed, as well as the VRP with time windows (VRPTW) with distance minimization. The proposed method compares very favorably with previous algorithms from the literature, and new improved solutions are reported for all considered problems.

1. Introduction

The *Vehicle Routing Problem* (VRP) and its variants have been the subject of considerable research efforts in the past year, mostly due to the growing number of additional constraints and objectives arising from real-world applications (Vidal, Crainic, Gendreau, & Prins, 2013a). Given the growing global concern about environmental issues, VRPs have recently started to incorporate "green" aspects such as pollution and alternative fuels, among others (see, e.g., the recent review of Lin, Choy, Ho, Chung, and Lam 2014 on VRPs with environmental issues).

Environmental costs due to greenhouse gas (GHG) emissions are not usually paid directly by the companies. Nevertheless, some countries are developing emissions trading schemes (e.g., the European Union Emissions Trading System) to make the companies responsible for their environmental impacts. The trend is that more and more countries will start to adopt emission-reducing actions, enhancing the importance of VRPs with environmental considerations. Furthermore, CO_2 is the most prominent transportation GHG, and its emission is closely related to fuel consumption (ICF, 2006).

In this work we turn our attention to the Pollution-Routing Problem (PRP), recently introduced by Bektaş and Laporte (2011). This problem seeks to jointly optimize travel itinerary and speed subject to time constraints, and thus poses significant methodological challenges due to the combination of two intricate sets of decisions. The PRP is $\mathcal{NP}$-hard since it includes the VRP with Time Windows (VRPTW) as a particular case, and most current exact procedures cannot address problems of practical sizes. A recent metaheuristic (Demir, Bektaş, & Laporte, 2012) has thus been proposed for the problem. This method is sequential, in the sense that routing decision variables are first optimized by solving a VRP with some fixed initial speeds and time windows, and speed optimization is only done during a post-optimization procedure. As highlighted by our computational experiments, this kind of approach may in some cases lead to solutions of lower quality.

To cope with this significant challenge, this paper introduces a new hybrid Iterated Local Search (ILS) for the PRP, specially designed to address the two families of decisions variables by means of adaptive travel time matrices, new tailored perturbation mechanisms and a Speed Optimization Algorithm (SOA) used at very specific steps of the search. This ILS follows the general structure

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of Subramanian, Penna, Uchoa, and Ochi (2012) and Subramanian, Uchoa, and Ochi (2013), and thus includes a further improvement procedure based on integer programming over a Set Partitioning (SP) formulation. Move evaluations are performed in amortized $\mathcal{O}(1)$ time by means of efficient incremental move evaluation procedures, and some infeasible solutions are used during shaking. Beyond its contribution on a key routing application, this paper proposes a methodology to jointly optimize both decision sets without necessarily relying on a systematic speed optimization during all move evaluations.

Extensive computational experiments demonstrate the high performance of the method on the existing PRP benchmark instances, characterized by large time windows, as well as on newer difficult instances with tighter time windows. We also consider a wider scope of application, demonstrating how two other "green" vehicle routing problems, the Fuel Consumption Vehicle Routing Problem (FCVRP; Xiao, Zhao, Kaku, & Xu, 2012) and the Energy Minimizing Vehicle Routing Problem (EMVRP; Kara, Kara, & Yetis, 2007) can be viewed as specific PRP instances. As a proof of concept, the proposed method is also tested on these problems, and on the VRPTW, leading to new state-of-the-art results.

The remainder of this paper is organized as follows. Section 2 presents some works that integrate VRPs with environmental aspects. Section 3 formally defines the PRP, EMVRP and FCVRP. Section 4 describes the proposed approach. Computational results are provided in Section 5, while Section 6 concludes.

2. Related works and challenges

Palmer (2007) was the first to incorporate environmental issues to the VRP. Different from the previous works that estimated the environmental costs based on the total duration or distance of the routes, the author considered other issues such as road topography, congestion and vehicle speeds to generate a CO_2 emissions matrix. Experiments suggested that the CO_2 minimization model compared with the distance-minimizing and duration-minimizing models led to a CO_2 reduction of 5.20 percent and 5.02 percent, respectively, on average.

Later on, Kara et al. (2007) proposed a mathematical formulation for the so-called *Energy Minimizing Vehicle Routing Problem* (EMVRP), which aims at minimizing the sum of the product between load and distance for each arc. Similar approaches, i.e., those that make use of the vehicle load to minimize the fuel consumption or CO_2 emissions, were presented by Peng and Wang (2009), Scott, Urquhart, and Hart (2010), Ubeda, Arcelus, and Faulin (2011) and Xiao et al. (2012). The latter introduced the *Fuel Consumption Vehicle Routing Problem* (FCVRP). Kara, Kara, and Yetis (2008) presented a VRP with cumulative costs, which generalizes the EMVRP as well as the *Minimum Latency Problem* and the *m-Traveling Repairman Problem*. Kopfer, Schnberger, and Kopfer (2014) took the load into account for CO_2 emissions evaluations in the presence of heterogeneous vehicle types.

Minimizing the fuel consumption considering only the load and distance can be insufficient since the travel speed plays a major role. This speed is directly affected by the road congestion. In view of this, Kuo (2010) proposed a model for minimizing the total fuel consumption where the speeds are time-dependent and the load is used to estimate the cost. A Simulated Annealing algorithm was implemented to solve a set of instances from the Solomon benchmark (Solomon, 1987). Later, Kuo and Wang (2011) devised a Tabu Search algorithm for the same problem. Other time-dependent problems with emission minimization can be found in Figliozzi (2011), Saberi and Verbas (2012) and Jabali, Van Woensel, and de Kok (2012), and many other references relevant to green logistics are mentioned in the surveys of Dekker, Bloemhof, and Mallidis (2012), Salimifard, Shahbandarzadeh, and Raeesi (2012), Lin et al. (2014) and Demir, Bektaş, and Laporte (2014b).

Bektaş and Laporte (2011) proposed the *Pollution-Routing Problem* (PRP), which seeks to minimize both operational and environmental costs, taking into account the customers time-window constraints. The total travel distance, the amount of load carried per distance unit, the vehicle speeds and the duration of the routes are the main cost components. Three different variants were also presented, considering either distance, weighted load and energy minimization. The authors performed an extensive experimental analysis to capture the trade-off between each variation, as well as the effect of the speed and time-window constraints on the distance, energy and costs.

The PRP was addressed with a two-phase heuristic in Demir et al. (2012). In the first phase, the VRPTW is solved by means of an Adaptive Large Neighborhood Search (ALNS), including five insertion operators and twelve removal operators. In a second phase, vehicle speeds are optimized using a recursive algorithm. Computational experiments were carried out for instances with up to 200 customers. Other recent developments consider generalizations of the problem. A bi-objective variant considering fuel and driving time minimization is presented in Demir, Bektaş, and Laporte (2014a), and Franceschetti, Honhon, van Woensel, Bektaş, and Laporte (2013) consider the time-dependent PRP.

It should be noted that several aspects of the PRP are conflicting. Higher speeds imply routes with shorter durations, but at the same time result in a larger amount of emissions and vice-versa. Hence, to reduce pollution, speed on arcs may be decreased to be closer to the speed which minimizes emissions. Yet with lower speed, the set of feasible VRP routes may be empty or drastically smaller. As a consequence, an optimal solution of the reduced speed VRP can have longer distance and even more emissions in some cases. One main goal of our study was to better integrate route and speed decisions in order to find quickly a suitable balance between these antagonist aspects.

3. Problem description

The PRP can be defined as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be a complete and directed graph with a set $\mathcal{V} = \{0, 1, 2, \dots, n\}$ of vertices and a set $\mathcal{A} = \{(i, j) \in \mathcal{V}^2, i \neq j\}$ of arcs. Vertex 0 represents the depot where a fleet of m identical vehicles with capacity Q is based. Vertices $\mathcal{V} - \{0\}$ correspond to customers, characterized by a non-negative demand q_i for a single product, a service time τ_i and a specified time-window interval $[a_i, b_i]$ for service. We assume that $q_0 = 0$ and $\tau_0 = 0$. Each arc $(i, j) \in \mathcal{A}$ represents a travel possibility from node i to j for a distance d_{ij}.

A particularity of the PRP is that the speed v_{ij} on each (i, j) is itself a decision variable, valued between $[v_{\text{MIN}}, v_{\text{MAX}}]$. Indeed, each vehicle emits on a certain amount of GHG which depends of weight and speed, among other factors. The PRP aims to find a speed matrix $(\mathbf{v})_{ij}$ and a set of routes $\mathbf{R}$ (such that $|\mathbf{R}| \leq m$) to serve all customers while minimizing environmental and operational costs. Each route $\boldsymbol{\sigma} \in \mathbf{R}$, $\boldsymbol{\sigma} = (\sigma_1, \dots, \sigma_{|\sigma|})$ starts and ends at the depot, i.e., $\sigma_1 = 0$ and $\sigma_{|\sigma|} = 0$, the total demand on each route should not exceed the vehicle capacity, and every customer must be visited within its time window.

Let $f_{\sigma_i \sigma_{i+1}}$ be the vehicle load on a route $\boldsymbol{\sigma}$ when traveling on arc (σ_i, σ_{i+1}). Let t_{σ_i} be the arrival time at customer σ_i knowing that each route starts at time zero, an early arrival to a customer i is permitted but triggers a waiting time to reach the start of the time window a_i, finally a late arrival is not permitted. The fuel consumption function presented in Bektaş and Laporte (2011) can be rewritten as Equation (1), where w_1, w_2, w_3, w_4 are parameters based on fuel properties, vehicle and network characteristics (Appendix A). Considering both the fuel consumption and the driving costs, the overall objective of the PRP is given in Equation (2).

$$F^{\text{F}}_{\sigma_i \sigma_{i+1}}(v_{\sigma_i \sigma_{i+1}}) = d_{\sigma_i \sigma_{i+1}} \left(\frac{w_1}{v_{\sigma_i \sigma_{i+1}}} + w_2 + w_3 f_{\sigma_i \sigma_{i+1}} + w_4 v^2_{\sigma_i \sigma_{i+1}} \right) \tag{1}$$