European Journal of Operational Research 000 (2015) 1–8

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

A fast calibrating volatility model for option pricing

Paresh Date*, Suren Islyaev

Center for the Analysis of Risk and Optimization Modelling Applications, Department of Mathematical Sciences, Brunel University, Middlesex UB8 3PH, UK

ARTICLE INFO

Article history:
Received 15 April 2014
Accepted 16 December 2014
Available online xxx

Keywords:
Stochastic volatility models
Option pricing

ABSTRACT

In this paper, we propose a new random volatility model, where the volatility has a deterministic term structure modified by a scalar random variable. Closed-form approximation is derived for European option price using higher order Greeks with respect to volatility. We show that the calibration of our model is often more than two orders of magnitude faster than the calibration of commonly used stochastic volatility models, such as the Heston model or Bates model. On 15 different index option data sets, we show that our model achieves accuracy comparable with the aforementioned models, at a much lower computational cost for calibration. Further, our model yields prices for certain exotic options in the same range as these two models. Lastly, the model yields delta and gamma values for options in the same range as the other commonly used models, over most of the data sets considered. Our model has a significant potential for use in high frequency derivative trading.

1. Introduction

The central assumption of the celebrated Black–Scholes formula for European option pricing is that the volatility of the underlying asset is constant (Black & Scholes, 1973). This is known to be untrue in practice. The observed prices of liquid options on the same underlying, for a given set of maturities and strikes, imply different volatilities under Black–Scholes formulation. Modelling the future evolution of the volatility of the underlying asset, which is consistent with the observed option prices, is obviously essential to price illiquid securities on the same underlying asset. The topic of suitable volatility models which provide a consistent match with the observed prices has resulted in extensive literature over the past few decades.

There are two broad classes of volatility models: local volatility models and stochastic volatility models. Note that this is a rather imprecise taxonomy, but it will be sufficient for our purpose. The former class of models does not have an additional source of uncertainty (apart from the sources of uncertainty in the underlying) incorporated in the volatility model and the volatility is assumed to be a deterministic function of the current underlying price and time. Examples of this type of models include the models proposed by Dupire (1994), Derman and Kani (1994) and (Alexander, 2004). In contrast, stochastic volatility models include an extra source (or sources) of randomness and provide more flexibility in modelling the dynamics of volatility surface. Significant models in this class, with an emphasis on option pricing, include those proposed by Hull and White (1987), Merton (1976), Heston (1993), Bates (1996), Kou (2002), Duffie, Pan, and Singleton (2000) and Carr, Geman, Madan, and Yor (2003). Bakshi, Cao, and Chen (1997) have compared a variety of stochastic volatility models in terms of their pricing and hedging performance. Heston as well as Bates model yields semi-closed form solutions in terms of Fourier transform of European option price and are hence amenable to relatively easy calibration to market data. Gatheral (2006) and Javaheri (2011) provide comprehensive reviews of development of volatility models.

In this work, we propose a new method for modelling the volatility as implied by the option prices. In our model, volatility is represented as a deterministic function of time, with its *level* being a random variable on positive support. The proposed volatility model offers the following benefits:

- It provides a very simple approximate pricing function for calibrating the model from option price data. In the experiments performed, we demonstrate that the proposed model requires only around 1 percent of the computational time as the Heston model or the Bates model for calibration, on the same hardware.
- In 15 different data sets tested for three different indices and using two different methods of measuring the pricing error, the proposed model is shown to be extremely competitive in terms of accuracy with the popular existing stochastic volatility models.
- When calibrated from the same data set, the proposed model also yields prices for path-dependent payoffs which are in the same range as the Heston model and Bates model. This is important since the prices of illiquid payoffs are non-unique under stochastic volatility and any new model which gives significantly different

* Corresponding author. Tel.: +44 1895 265613; fax: +44 1895 269732.
E-mail address: paresh.date@brunel.ac.uk (P. Date).

http://dx.doi.org/10.1016/j.ejor.2014.12.031

Please cite this article as: P. Date, S. Islyaev, A fast calibrating volatility model for option pricing, European Journal of Operational Research (2015), http://dx.doi.org/10.1016/j.ejor.2014.12.031

prices from the established models is unlikely to be accepted by the industrial community.

- When calibrated from the same data set and using the same numerical method, the proposed model yields option price sensitivity parameters which are very close to those found for one of the two benchmark models, for most data sets. Option sensitivities (or Greeks) are important for risk monitoring and hedging purposes and our experiments show that hedging using our model is unlikely to provide significantly different results than hedging using the Heston model.

Note that, apart from the Bates model and the Heston model, several other analytically tractable options exist for modelling volatility (as mentioned earlier). Our purpose is simply to establish that our new model yields accuracy comparable to some of the popular existing models, while being significantly easier to calibrate, and easier to simulate from, than those models. Hence we have restricted our benchmark comparison to the two aforementioned models.

The rest of the paper is organized as follows. In the next section, we will briefly outline the two main stochastic volatility models to which our model will later be compared. In Section 3, we will present our new model. Section 4 on numerical experiments is split into three subsections: Section 4.1 outlines the data used, Section 4.2 explains the methodology employed in comparing the performance of different models and lastly Section 4.3 provides the results and a discussion. Finally, Section 5 summarizes the contributions of the paper and outlines the directions of future research.

2. Heston model and Bates (SVJ) model

We will first outline the formulae for pricing European options using Heston and Bates (SVJ) models, since we will later use these two models as benchmarks. All the subsequent discussion is in a (non-unique) equivalent martingale measure and we will omit explicit mention of measure for simplicity. For the Heston model, the asset price dynamics is assumed to be governed by:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^1, \tag{1}$$

$$dv_t = -\theta(\bar{v} - v_t)dt + \sigma_v \sqrt{v_t} dW_t^2, \tag{2}$$

where r is the risk-free rate, W_t^1 and W_t^1 are standard Wiener processes with a given correlation $\langle W_t^1, W_t^2 \rangle = \rho$ and $\rho, \sigma_v, \theta, v_0, \bar{v}$ are known constants. The price of European call option with strike price K is given by:

$$C_{\mathrm{EUR}} = S_t P_1 - Ke^{-r(T-t)} P_2, \tag{3}$$

where S_t is the spot price at time t, T is the expiration time and $P_j, j = 1, 2$ are called the pseudo-probabilities:

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re\left[\frac{e^{ix\log(\frac{S_t}{K})} e^{\phi_j(v_t,\tau,x)}}{ix}\right] dx. \tag{4}$$

Here, $\tau = T - t$ and $\phi_j(v_t, \tau, x) = \exp\{C_j(\tau, x)\bar{v} + D_j(\tau, x)v_t\}$ is the characteristic function, with

$$C_j(\tau, x) = rxi\tau + \frac{\theta}{\sigma_v^2}\left[(b_j - \rho\sigma_v xi + d_j)\tau - 2\log\frac{1 - d_j e^{d_j\tau}}{1 - g_j}\right],$$

$$D_j(\tau, x) = \frac{b_j - \rho\sigma_v xi + d_j}{\sigma_v^2}\left[\frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}}\right],$$

$$g_j = \frac{b_j - \rho\sigma_v xi + d_j}{b_j - \rho\sigma_v xi - d_j},$$

$$d_j = \sqrt{(\rho\sigma_v xi)^2 - \sigma_v^2(2u_j xi - x^2)},$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, \text{ and } b_j = \kappa + \theta - (\mathbb{1}_{j=1})\rho\sigma_v.$$

Bates (1996) proposed adding a compound Poisson process in the underlying for the above model, which leads to a modification of (1):

$$\frac{dS_t}{S_t} = rdt + \sqrt{v_t} dW_t^1 + (e^{\alpha+\beta\epsilon} - 1)dJ_t, \tag{5}$$

where J_t is Poisson process with a known jump intensity λ_p, α, β are known constants and $\epsilon \sim N(0, 1)$. The process J_t is uncorrelated with W_t^i, $(i = 1, 2)$. The volatility dynamics is described by Eq. (2). This model is also commonly referred to as SVJ (stochastic volatility with jumps) model. The solution for price of a European call option is given by modifying the characteristic function in the Heston model above:

$$\phi_j(v_t, \tau, x) = exp\{C_j(\tau, x)\bar{v} + D_j(\tau, x)v_t + E(x)\tau\},$$

where

$$E(x) = -\lambda_p ix(e^{\alpha+\beta^2/2} - 1) + \lambda_p(e^{ix\alpha - x^2\beta^2/2} - 1).$$

While both these models have proved popular and are known to provide good fits to option prices, they have a few shortcomings. Some of these are discussed in Mikhailov and Nögel (2004). In particular, it was shown that Heston model usually fails to fit to a short term market skew while the SVJ model usually fails to fit an inverse yield curve. In addition, the option price is given through a fairly involved numerical integral with several parameters, which presents significant difficulties in calibration.

3. High order moments based stochastic volatility model

We will now introduce the basic idea of our model. Recall that, by definition, European call option is a right to buy an asset at maturity time T for a strike price K. For a non-dividend paying stock, its price at time t is given by discounted expectation of terminal pay-off:

$$C_t = e^{-r(T-t)}\mathbb{E}[(S_T - K, 0)^+].$$

Under Black–Scholes framework with constant volatility, this discounted expected value is given by

$$\begin{aligned} C_{\mathrm{BS}} &= S_t N(d_1) - e^{-r\tau} K N(d_2), \\ d_1 &= (\sigma\sqrt{\tau})^{-1}[\log(S_t/K) + (r + \sigma^2/2)\tau], \\ d_2 &= d_1 - (\sigma\sqrt{\tau}), \end{aligned}$$

where r is the constant risk-free rate, σ is the volatility, $N(x)$ is the standard normal cumulative distribution function and $\tau = T - t$ is the time to maturity. The derivation of Black–Scholes price also assumes that short-selling as well as trading in continuous time is possible. One of the simplest frameworks to introduce a stochastic component in the volatility is to consider a Hull–White type model of the asset price process (Hull & White, 1987):

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^1, \tag{6}$$

$$dv_t = f_1(t, v_t)dt + f_2(t, v_t)dW_t^2, \tag{7}$$

where W_t^1 and W_t^2 are uncorrelated Wiener processes and f_1, f_2 are smooth functions bounded by linear growth such that v_t remains non-negative almost surely. Hull and White (1987) show that the price of European vanilla call option at time 0, for a time to maturity τ can be derived as expectation of Black–Scholes price with respect to the variance rate:

$$C_{\mathrm{EUR}} = \mathbb{E}\left[C_{\mathrm{BS}}\left(\frac{1}{\tau}\int_0^\tau v_t dt\right)\right] \tag{8}$$

where $C_{\mathrm{BS}}(x)$ denotes Black–Scholes price evaluated at variance x. The above formula is independent of the exact process followed by v_t (under normal assumptions about $t-$ continuity and uniqueness). Denoting the variance rate $\frac{1}{\tau}\int_0^\tau v_t dt$ by $\bar{V}_\tau$ and assuming that the moments in question exist, we can expand the right hand side of (8)