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The Center of a Convex Set and Capital Allocation

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Research Highlights

- A capital allocation scheme for coherent risk measures has been suggested.
- It returns the *unique* solution for *every* coherent risk measure.
- The resulting capital allocation is linear and diversifying.
- The method can be applied to fair division in optimal risk sharing problems.

Abstract

A capital allocation scheme for a company that has a random total profit Y and uses a coherent risk measure ρ has been suggested. The scheme returns a *unique* real number $\Lambda^*_\rho(X,Y)$, which determines the capital that should be allocated to company's subsidiary with random profit X. The resulting capital allocation is linear and diversifying as defined by Kalkbrener (2005). The problem is reduced to selecting the "center" of a non-empty convex weakly compact subset of a Banach space, and the solution to the latter problem proposed by Lim (1981) has been used. Our scheme can also be applied to selecting the unique Pareto optimal allocation in a wide class of optimal risk sharing problems.

Key Words: capital allocation, risk contribution, coherent risk measures, risk sharing.

1 Introduction

One of the basic problems in risk management is to determine the allocation of risk capital among agents or business units. We make two assumptions: (i) a company consists of n subsidiaries, each contributing a random profit X_i, so that the total profit of the company is $Y^* = \sum_{i=1}^n X_i$, and (ii) the company has decided, or is required by a regulator, to reserve a risk capital $\rho(Y^*)$, to compensate possible loss, where $\rho(.)$ is a fixed risk measure. The capital allocation problem is to distribute $\rho(Y^*)$ among subsidiaries, that is, to assign subsidiary i the capital k_i with $\sum_{i=1}^n k_i = \rho(Y^*)$. The numbers k_i are called *risk contributions* of X_i to the Y^*.

If $\rho(Y^*) \leq \sum_{i=1}^n \rho(X_i)$, it is possible to find a capital allocation such that $k_i \leq \rho(X_i), i = 1,\dots,n$. This corresponds to the intuition of diversification: the risks produced by subsidiaries partially compensate each other, which allows one to reduce the risk contribution of each of them. If $\rho(Y^*) = \sum_{i=1}^n \rho(X_i)$, such an allocation is unique and given by $k_i = \rho(X_i), i = 1,\dots,n$. The capital allocation problem can be formulated as follows.

Problem I *Assume that $\rho(Y^*) < \sum_{i=1}^n \rho(X_i)$, so that a capital allocation satisfying $k_i \leq \rho(X_i), i = 1,\dots,n$ is not unique. Which one to choose?*

The capital allocation problem in this or similar form has been extensively studied in a number of papers, see eg. Denault (2001), Fisher (2003), Delbaen (2004), Kalkbrener (2005), Cherny and Orlov (2011) and references therein. We rely on a natural assumption (see Kalkbrener (2005)), that the risk contribution k_i of subsidiary i depends only on X_i and Y^*, but not on the decomposition of $Y^* - X_i$ among the rest of subsidiaries. In this context, a capital allocation with respect to risk measure $\rho(.)$ is just a function of two arguments $\Lambda_\rho(X,Y)$, such that $\Lambda_\rho(Y,Y) = \rho(Y)$. With $k_i = \Lambda_\rho(X_i, Y^*)$, the requirements (i) $\sum_{i=1}^n k_i = \rho(Y^*)$ and (ii) $k_i \leq \rho(X_i), i = 1,\dots,n$ can now be rewritten as

(i) (Linearity) $\Lambda_\rho(X,Y)$ is a linear functional in the first argument;