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Cooperation on capacitated inventory situations with fixed holding costs

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ABSTRACT

In this paper we analyze a situation in which several firms deal with inventory problems concerning the same type of product. We consider that each firm uses its limited capacity warehouse for storing purposes and that it faces an economic order quantity model where storage costs are irrelevant (and assumed to be zero) and shortages are allowed. In this setting, we show that firms can save costs by placing joint orders and obtain an optimal order policy for the firms. Besides, we identify an associated class of costs games which we show to be concave. Finally, we introduce and study a rule to share the costs among the firms which provides core allocations and can be easily computed.

1. Introduction

The analysis of multi-agent inventory models is a flourishing research field in the frontier between game theory and operations research. In a multi-agent inventory model several agents facing individual inventory problems cooperate by coordinating their orders for the purpose of reducing costs. In the analysis of one of these models two main issues are usually addressed: first, what is the optimal order policy of the group of cooperating agents; second, how the ordering costs should be shared among the agents. Meca, García-Jurado, and Borm (2003) focus on a joint replenishment problem where agents follow an *Economic Production Quantity* policy with shortages. Meca, Timmer, García-Jurado, and Borm (2004) study the joint replenishment problem where agents agree to place joint orders by means of the classical *Economic Order Quantity* policy. In both papers authors use cooperative games to model the corresponding situations. Besides, in Meca et al. (2003) and Korpeoglu, Sen, and Guler (2012) a non-cooperative approach is taken. Nagarajan and Sošić (2008), Dror and Hartman (2011) and Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2012) are recent surveys of multi-agent inventory models; Fiestras-Janeiro, García-Jurado, and Mosquera (2011) review the applications of cooperative game theory for sharing cost problems.

In most inventory models a positive storage cost per item and time unit is assumed to exist. However, in some situations storage costs are fixed (i.e. independent of the size of the stock) and therefore can be disregarded in the optimization problem. This can be the case, for instance, when the storage costs are only due to the maintenance of the warehouse. Notice that when storage costs are irrelevant and fixed ordering costs are positive, in a continuous review setting, the orders should be as large as possible and, thus, the capacity of the warehouse becomes significant.

There are many papers dealing with limited capacity inventory models. In fact, most of the classical and modern books on inventory management include the basic ideas on capacitated inventory; see, for instance, Tersine (1994) and Zipkin (2000). A survey on capacitated lot sizing can be found in Karimi, Fatemi Ghomi, and Wilson (2003). More recently, Ng, Cheng, Kotov, and Kovalyov (2009) study an economic order quantity model where the warehouse capacity is limited and is, moreover, a decision variable of the model. Parker and Kapuscinski (2011) consider the non-cooperative interaction between a retailer and a supplier in a two-stage, periodic review, limited capacity inventory model; it provides a Markov equilibrium policy for the model. On the contrary, as far as we know, apart from Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2014), the literature has not treated multi-agent inventory models with limited capacity and fixed storage costs. However, there are a variety of real situations which may be modeled in this way.

Fiestras-Janeiro et al. (2014) deal with an inventory problem arising in a farming community in the Northwest of Spain. It considers a collection of stockbreeders (each one owning a relatively small livestock farm) that need livestock feed and place orders to an external supplier. Each farm has its own *silo* (warehouse), with limited capacity, for keeping the feed. The only costs associated with the silos are

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their building costs since their maintenance costs are irrelevant; thus, the storage cost of each stockbreeder is in fact zero. Fiestras-Janeiro et al. (2014) analyze then two models with n decision makers, all them facing continuous review inventory problems without holding costs, with limited capacity warehouses and without shortages. The fact that shortages are not allowed simplifies strongly the search for optimal policies. However, the case with shortages can be also used in this context, as we discuss in Example 4.1.

In this paper we analyze a situation in which several firms deal with inventory problems concerning the same type of product and cooperate by placing joint orders. We consider that each firm uses its limited capacity warehouse for storing purposes and that it faces an economic order quantity model where storage costs are irrelevant (and assumed to be zero) and shortages are allowed. To illustrate our results we use the example in Fiestras-Janeiro et al. (2014) when shortages are allowed. However, the model we introduce in this paper can be successfully used in other examples, like the following one. Farmers often have their own farm tractors. The fuel for these vehicles is commonly stored in tanks at the farms, with no maintenance costs. If a farm's tank is depleted, then the farm can borrow some extra fuel from one of its neighbors, at a small cost. Nevertheless, when this farm makes its new fuel order, it has to order an extra amount of fuel, enough for restoring the borrowed fuel. The replenishment problem when several farms cooperate can be analyzed with the tools developed in this paper.

The organization of this paper is as follows. First we introduce the model we analyze: economic order quantity systems without holding costs. Then we deal with the one decision maker case, and later we study the case with n firms. We show that firms can save costs by placing joint orders and, in this case, we obtain an optimal order policy for the firms. Finally, we provide some results that can be helpful for allocating the joint costs among the firms.

2. The model

An *EOQ (Economic Order Quantity) system without holding costs* is a multi-agent situation where each agent faces a continuous review inventory problem with no holding costs, with shortages and with a limited capacity warehouse. We assume that the lead time is deterministic and can be taken as zero. N denotes the finite set of agents. The parameters associated to every $i \in N$ in an EOQ system without holding costs are:

- $a > 0$, the fixed cost per order,
- $b_i > 0$, the shortage cost per item and per time unit,
- $d_i > 0$, the deterministic demand per time unit,
- $K_i > 0$, the capacity of i's warehouse.

As we mentioned in the Introduction, this model is in fact a generalization of one introduced in Fiestras-Janeiro et al. (2014): the basic EOQ system without holding costs. These basic systems do not allow for shortages and, then, the analysis of the model we introduce in this paper is fully different. In the operation of an EOQ system without holding costs, every time that agent i's maximum shortage level is reached, agent i places an order of size $K_i + \beta_i$ (since the storage cost is zero, agent i's warehouse should be complete after each order). Nevertheless, every agent i has to make a decision on his maximum shortage level β_i because K_i is fixed. The interval time between two consecutive orders of agent i is called a cycle and its length is $\frac{K_i+\beta_i}{d_i}$. In an agent i's cycle, the length of the period that he incurs into shortages is $\frac{\max\{\beta_i,0\}}{d_i}$. Besides, taking into account that the demand is deterministic, the average shortage level in the shortage period during a cycle is $\frac{\max\{\beta_i,0\}}{2}$. Then, agent i's average cost per cycle is given by

$$a + b_i \frac{\max\{\beta_i,0\}}{2}\frac{\max\{\beta_i,0\}}{d_i}$$

and agent i's average cost per time unit is given by

$$C^i(\beta_i) = \frac{a + \frac{b_i \max^2\{\beta_i,0\}}{2d_i}}{\frac{K_i+\beta_i}{d_i}} = \frac{ad_i}{K_i+\beta_i} + \frac{b_i \max^2\{\beta_i,0\}}{2(K_i+\beta_i)},$$

where $\beta_i > -K_i$ in order to guarantee a positive cycle length.[1] We rewrite the agent i's cost function as

$$C^i(\beta_i) = \begin{cases} \dfrac{ad_i}{K_i+\beta_i} & \text{if } -K_i < \beta_i \le 0 \\ \dfrac{ad_i}{K_i+\beta_i} + \dfrac{b_i\beta_i^2}{2(K_i+\beta_i)} & \text{if } 0 \le \beta_i. \end{cases}$$

For simplicity we take the number of orders per time unit as the decision variable, that is

$$x_i := \frac{d_i}{K_i+\beta_i}, \tag{1}$$

which implies that

$$\beta_i = \frac{d_i - K_i x_i}{x_i}.$$

Then agent i's cost function can be written as

$$C^i(x_i) = \begin{cases} ax_i & \text{if } x_i \ge \frac{d_i}{K_i} \\ ax_i + \dfrac{b_i(d_i-K_ix_i)^2}{2x_id_i} & \text{if } 0 < x_i \le \frac{d_i}{K_i}. \end{cases} \tag{2}$$

Observe that the ratio demand/capacity (d_i/K_i) is present in Expression (2). It will play a relevant role in other issues regarding this model as we will see later on, especially in Section 5.

In this paper we explore the possibilities of cooperation in an EOQ system without holding costs. When we look at this model from a cooperative point of view, we consider that a non-empty coalition $S \subset N$ has formed and assume that all its members place joint orders. It means that the cycle length will be the same for every agent in S, i.e.

$$\frac{1}{x_i} = \frac{K_i+\beta_i}{d_i} = \frac{K_j+\beta_j}{d_j} = \frac{1}{x_j}, \tag{3}$$

for every $i, j \in S$. Equivalently, $x_i = x_j$ for every $i, j \in S$, i.e., the number of orders per time unit will be the same for every agent in S. For simplicity we denote $x = x_i$ for every $i \in S$. Now, the average cost per cycle that coalition S faces is given by

$$a + \sum_{i\in S} b_i \frac{\max\{\beta_i,0\}}{2}\frac{\max\{\beta_i,0\}}{d_i}$$

and the average cost per time unit is given by

$$\frac{a + \sum_{i\in S}\frac{b_i \max^2\{\beta_i,0\}}{2d_i}}{\frac{K_j+\beta_j}{d_j}} = \frac{ad_j}{K_j+\beta_j} + \frac{d_j}{K_j+\beta_j}\sum_{i\in S}\frac{b_i\max^2\{\beta_i,0\}}{2d_i}.$$

Using the condition of equal cycle length (3), we have that $\beta_i = -K_i + \frac{d_i}{x}$ for all $i \in S$, being x the number of orders per time unit. Thus, for every $x > 0$, the average cost per time unit of coalition S is given by

$$\begin{aligned} C^S(x) &= ax + x\sum_{i\in S}\frac{b_i}{2d_i}\max^2\left\{-K_i + \frac{d_i}{x}, 0\right\} \\ &= ax + \frac{1}{x}\sum_{i\in S}\frac{b_i}{2d_i}\max^2\{-K_ix + d_i, 0\}. \end{aligned} \tag{4}$$

[1] In principle, each β_i is non-negative. However, when a group of agents places joint orders it may be optimal that the maximum shortage level of some agents is negative; notice that in our context storage costs are irrelevant.