European Journal of Operational Research 241 (2015) 412–421

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Optimum step-stress accelerated degradation test for Wiener degradation process under constraints

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ARTICLE INFO

Article history:
Received 28 May 2013
Accepted 8 September 2014
Available online 22 September 2014

Keywords:
Reliability
Accelerated Degradation Test
Step-Stress ADT
Wiener process
Inverse Gaussian distribution

ABSTRACT

To assess a product's reliability for subsequent managerial decisions such as designing an extended warranty policy and developing a maintenance schedule, Accelerated Degradation Test (ADT) has been used to obtain reliability information in a timely manner. In particular, Step-Stress ADT (SSADT) is one of the most commonly used stress loadings for shortening test duration and reducing the required sample size. Although it was demonstrated in many previous studies that the optimum SSADT plan is actually a simple SSADT plan using only two stress levels, most of these results were obtained numerically on a case-by-case basis. In this paper, we formally prove that, under the Wiener degradation model with a drift parameter being a linear function of the (transformed) stress level, a multi-level SSADT plan will degenerate to a simple SSADT plan under many commonly used optimization criteria and some practical constraints. We also show that, under our model assumptions, any SSADT plan with more than two distinct stress levels cannot be optimal. These results are useful for searching for an optimum SSADT plan, since one needs to focus only on simple SSADTs. A numerical example is presented to compare the efficiency of the proposed optimum simple SSADT plans and a SSADT plan proposed by a previous study. In addition, a simulation study is conducted for investigating the efficiency of the proposed SSADT plans when the sample size is small.

1. Introduction

Continued advances in manufacturing technology, coupled with consumer desire for high quality products, have prompted the industry to design and manufacture products that can operate without failure for years. However, for such highly reliable products, it is not an easy task to assess the product reliability within short test durations because sufficient lifetime data are generally required to precisely estimate product's lifetime (failure time, time-to-failure) distribution. Precise reliability estimation is an important part of subsequent managerial decisions such as determining the burn-in time (Sheu & Chien, 2005; Tsai, Tseng, & Balakrishnan, 2011; Ye, Shen, & Xie, 2012), establishing a warranty or maintenance policy (Chien, 2008; Jung & Park, 2003), or pricing extended warranties. To increase the likelihood of observing failures, Accelerated Life Test (ALT) is commonly used by exposing and testing products under a higher stressed condition (e.g., higher temperature, voltage, pressure, vibration, electric current, etc.). Seo, Jung, and Kim (2009) proposed accelerated life test sampling plans satisfying producer's and consumer's risk requirements for deciding the lot acceptability. However, for many highly reliability products, it might still be difficult to obtain enough failure data with short test duration even if an ALT is used. For products like these, an Accelerated Degradation Test (ADT) provides an alternative effective tool to estimate the lifetime. ADT has been successfully applied to many modern products like Light-Emitting Diodes (LED), as in the study by Pan and Crispin (2010).

In an ADT, units are exposed to a relatively severe environment. However, in addition to collecting exact failure time data, measurements of a certain product Quality Characteristic (QC) are recorded at various inspection times. The QC usually degrades (or increases) over time and the lifetime of the product is normally related to the level of the QC. For example, the life of an alloy can be defined when its crack (the QC) reaches size 1.6 inches (Meeker & Escobar, 1998). The life of a certain self-regulating heating cable is related to its resistance (Whitmore & Schenkelberg, 1997). For some elastomers, which are critical materials for hoses and dampers, the life is related to its hardness measure (Elsayed, 2012).

Since the QC of a product degrades over time, the product's life can then be defined as the first-passage time when the QC crosses a

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http://dx.doi.org/10.1016/j.ejor.2014.09.003
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pre-specified threshold. ADT data normally consists of measurements of the QC at each measuring time under different stress levels and possible failure times. After ADT data is collected, a statistical model is required for analyzing the observed degradation data and to estimate the product's lifetime under use condition. There are many different models in the literature for fitting a degradation path; for example, the mixed effects nonlinear regression model (Lu & Meeker, 1993; Zhou, Gebraeel, & Serban, 2012), the Gamma process model (Tsai et al., 2011; Tseng, Balakrishnan, & Tsai, 2009), the inverse Gaussian process model (Wang & Xu, 2010), the linear and exponential-based degradation model (Si, Wang, Chen, Hu, & Zhou, 2013), and the Wiener process model (Doksum & Hoyland, 1992; Lee & Tang, 2007). Si, Wang, Hu, and Zhou (2011) provided a review on many stochastic models for estimating the remaining useful life. In this paper, we consider the Wiener process to model the degradation path of a product's QC. Under this assumption, it is well-known that the product's life follows an Inverse Gaussian (IG) distribution. The Wiener/IG model has been used for many applications in a variety of studies. For example, Sherif and Smith (1980) and Bhattacharyya and Fries (1982) considered a fatigue failure model in which accumulated decay is governed by a Wiener process. Doksum and Hoyland (1992) used a time-transformed Wiener process to model an accelerated degradation sample path. Doksum and Normand (1995) assumed biomarker processes such as calibrated log CD4 blood cell counts were Wiener processes in their HIV study. Whitmore and Schenkelberg (1997) also used a time-transformed Wiener process to model resistance of self-regulating heating cables. Elwany and Gebraeel (2009) used a Brownian motion with positive drift and the IG lifetime distribution to obtain a conservative estimate of an operating component's mean remaining life for subsequent managerial decisions. For other research efforts using the Wiener degradation model, see, for example, Whitmore, Crowder, and Lawless (1998), Padgett and Tomlinson (2004), Tseng, Tang, and Ku (2003), Tseng and Peng (2004), Balka, Desmond, and McNicholas (2009), Gebraeel, Lawley, Li, and Ryan (2005), Park and Padgett (2005), Lehmann (2009), and Wang (2010).

Before an ADT is conducted, one needs to decide how the stress level (stress loadings) should be increased or decreased. Several types of stress loading have been proposed in the literature including Constant-Stress ADT (CSADT) and Step-Stress ADT (SSADT). For a CSADT, Yu and Tseng (1998) proposed a stopping rule for terminating a degradation test. Park and Yum (1997) developed plans in which they determined the stress levels, the proportion of units allocated to each stress level, and the measurement times such that the asymptotic variance of the Maximum Likelihood Estimate (MLE) of the mean lifetime at the use condition is minimized. Typically, a test plan is designed so that a precise estimate can be obtained. For other references about CSADT, see Meeker and Escobar (1998), Yu (2003, 2006), Wu and Chang (2002), and Lim and Yum (2011).

Recently, researchers have considered using some time-varying stress loadings in order to further shorten test duration and reduce the number of test devices and the sample size. SSADT is a special type of stress loading in which all units are tested together and the stress level is increased step-wisely. When there are only two stress levels (i.e., one change), the test plan is often referred to as a *simple* SSADT plan. SSADT is commonly used because it is often easier to administer than a general time-varying stress plan and has the advantage that only a few test units are needed. It has been shown that using step-stress stress loading can provide equivalent estimation precision to that from other stress loadings (see Hu, Plante, & Tang, 2013; Liao & Elsayed, 2010). Further, Lee and Tang (2013) have shown that there exist SSADT plans that can generate a Fisher information matrix identical to that derived from a general stress loading function. Given these advantages of using SSADT, extensive studies have been conducted to obtain optimum SSADT plans. For example, Tang, Yang, and Xie (2004) designed a SSADT to minimize the total expected test cost, which is a function of sample size, test duration, and the number of inspections. Liao and Tseng (2006) provided SSADT plans to minimize the variance of estimated p-percentile under a budget constraint. Recently, Tseng et al. (2009) introduced a SSADT plan minimizing the approximate variance of the estimated MTTF when the degradation path follows a gamma process. Zhang, Jiang, Li, and Wang (2010) and Ge, Li, Jiang, and Huang (2011) have also provided algorithms to obtain SSADT plans for several different objectives. For an overview of degradation test models, as well as design problems, refer to Boulanger and Escobar (1994), Meeker and Escobar (1998), Nelson (2005a, 2005b), and Yum, Lim, and Seo (2007).

Although extensive research efforts, including the articles cited above, have been devoted to obtaining optimal SSADT plans, most of their results are based primarily on numerical studies. It is important to mention that, in many of these research efforts, the numerical results suggest that the optimum SSADT design is actually a simple SSADT using only the minimum and maximum stress levels even when their objectives are quite different. In this paper, we formally show that this result holds for many commonly used objective functions. Secondly, when focusing on designing a simple SSADT, the optimal allocations of inspection efforts are derived under various optimization criteria. The remainder of this paper is organized as follows. In Section 2, we describe the accelerated degradation model used in this paper and introduce the decision variables and constraints considered in designing a SSADT plan. In Section 3, we derive the MLEs of the model parameters and the Fisher information matrix. Using this matrix, we show that for several commonly used objective functions, the optimal SSADT plan is indeed a simple one. Optimal simple SSADT plans are then derived in Section 4. In Section 5, a numerical example is provided to compare the efficiency of a SSADT plan proposed by a previous study and the optimum plans proposed in this paper. A simulation study is also conducted for investigating the efficiency of optimum SSADT plans when the sample size is small. Finally, concluding remarks and possible directions for future study are given in Section 6.

2. The model and design for a SSADT

In this section, we introduce the notations and the model assumptions used throughout this paper. Decision variables, as well as constraints, for designing a SSADT plan are also discussed.

2.1. A SSADT plan

2.1.1. Decision variables

In a SSADT, all test units are exposed to an initial stress level (denoted by s_1) and tested independently until a pre-specified stress change time. The stress is then adjusted to another level (denoted by s_2) for the surviving test units. There may be more than one stress adjustment before the test is terminated. Under each stress level, s_i, the surviving units are inspected and the degradation increments are recorded at pre-specified time points $t_{ij}, j = 1, 2, \ldots, l_i, i = 1, 2, \ldots, k$ where k is the total number of stress levels and l_i is the number of inspections under s_i. We assume that under all stress levels, inspections are conducted at the same inspection time interval, Δt. Hence, the total test duration under s_i is $l_i \times \Delta t$. In this paper, we assume that the sample size (N), the inspection interval (Δt), and the stress levels ($s_i, i = 1, 2, \ldots, k$) are pre-specified and the decision variables are the number of inspections under each stress level (i.e., $l_i, i = 1, 2, \ldots, k$) when optimizing several of the commonly used objective functions described in Section 3.

2.1.2. Constraints in SSADT planning

To shorten the test duration, one of the most commonly used constraints is the time constraint. That is, a SSADT is terminated at a pre-specified time, since the total test budget is often limited in practice. The budget does affect not only the test duration but also the