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Entrepreneurial finance with equity-for-guarantee swap and idiosyncratic risk ☆

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ABSTRACT

We consider a risk-averse entrepreneur who invests in a project with idiosyncratic risk. In contrast to the literature, we assume the entrepreneur is unable to get a loan from a bank directly because of the low creditability of the entrepreneur and so an innovative financial contract, named equity-for-guarantee swap, is signed among a bank, an insurer, and the entrepreneur. It is shown that the new swap leads to higher leverage, which brings more diversification and tax benefits. The new swap not only solves the problems of financing constraints, but also significantly improves the welfare level of the entrepreneur. The growth of welfare level increases dramatically with risk aversion index and the volatility of idiosyncratic risk.

1. Introduction

One of fundamental characteristics of entrepreneurship is lack of diversification. Specifically, the revenue of an entrepreneurial firm (private firm) suffers systematic and idiosyncratic risks. Entrepreneurs can trade risk-free bonds and the diversified market portfolio to diversify the systematic business risk but not the idiosyncratic risk. Therefore, the diversification benefit of risky debt is important to entrepreneurs in addition to the standard trade-off between tax benefits and costs of financial distress, see Chen, Miao, and Wang (2010) among others.

In addition, there are many small and medium enterprises (SMEs) and fresh graduates every year who are hungry for money to start a new business. Such investment is generally extremely high-risk, and to compensate for such risk, the entrepreneur comes with the potential for high returns. However, due to low credibility and lack of guarantee, many entrepreneurs, let alone fresh graduates, are unable to get a bank loan or get other debt financing cheaply. Under such situation, traditional financial theory on optimal capital structure is not reasonable since the entrepreneur has no other choice beyond starting her business with her own money only or simply giving up the business.

To overcome borrowing constraints, some insurers and entrepreneurs in China have developed an innovative financial product, called equity-for-guarantee swap (EGS). This is an agreement between a lender (bank), an insurer, and a borrower (entrepreneur), where the bank lends at a given interest rate to the entrepreneur and if the entrepreneur defaults on the debt, the insurer will make a compensatory payment to the creditor so that the creditor will always be paid up-to a certain guarantee level. In return for the guarantee, the firm needs to allocate a percentage of the firm's equity to the insurer. This contract was first signed in 2002 in China and it has become increasingly popular in the country.[1]

In this article, we extend the model established by Chen et al. (2010) to take into account both idiosyncratic risk and the EGS. This paper relates to Yang and Zhang (2013), who provide the first formal study on the swap. However, Yang and Zhang (2013) merely discuss traditional capital structure issues in the classic framework of Leland (1994). Our model examines this contract in a more general context with idiosyncratic risk and cash-out option.

The main results in Chen et al. (2010) are based on the assumption that the entrepreneur has "deep pockets", i.e. she can issue debt with the coupon rate being higher than the project's revenue since

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[1] The guarantee in our model shares a few similarities with that in Ju and Sohn (2014), where the contract is based on a technology credit scoring model.

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she can inject cash into the firm to pay coupons. However, this assumption is not feasible for many entrepreneurs, not to mention fresh graduates. Actually, Chen et al. (2010) point out that entrepreneurs may be liquidity-constrained, i.e. no external funds are available to cover the firm's debt service, and hence an earlier liquidation will be forced by the creditor. We argue that the assumption becomes practical thanks to the EGS. In fact, under the swap, the entrepreneur is equivalent to the one who has deep pockets and the default threshold can be lower than the coupon level because the claim owned by the creditor is guaranteed by the insurer. In exchange for the guarantee, the entrepreneur needs to pay the insurer a proportion of equity of the firm. In addition, since the insurer guarantees the debt, the creditor under the swap does not demand a protective covenant.

We consider a risk-averse entrepreneur having access to standard financial investment opportunities with a chance to invest in a project. The objective of the entrepreneur is to maximize her expected lifetime utility over intertemporal consumption. We choose the exponential utility primarily for analytical tractability. While constant absolute risk aversion (CARA) utility does not capture wealth effects, it reduces the dimension, especially for the double-barrier boundary problem, see Henderson (2002), Miao and Wang (2007), Ewald and Yang (2008), and Yang and Yang (2012) among others.[2]

The main results of the paper are as follows. First, our setting improves a generalized model of capital structure trade-off among borrowing constraints, tax, diversification benefits, and costs of financial distress. Second, the EGS fundamentally raises the entrepreneur's borrowing capacity and therefore the entrepreneur optimally issues more debt and takes higher leverage than that without the swap. Higher leverage leads to larger tax shields and diversification benefits because the entrepreneur faces less equity exposure to the project and thus her portfolio is less risky. Third, the entrepreneur with the swap receives more welfare increments and has more investment opportunities because of being more willing to invest. Higher risk-averse entrepreneurs under higher non-diversifiable idiosyncratic risk gain more benefits resulting from the swap.

This paper is organized as follows. Section 2 presents the model. Section 3 solves the model. Section 4 discusses the numerical results. Section 5 concludes. Appendices provide equilibrium valuation of corporate securities.

2. Model setup

2.1. Investment opportunities

We consider an infinitely-lived risk-averse entrepreneur who has an option to invest in a take-it-or-leave-it project at present time 0, which requires a one-time investment cost I. All sources of uncertainty arise from two independent standard Brownian motions B and Z defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathbb{P})$.

In addition to the project opportunity, the entrepreneur has access to standard financial investment opportunities, see Merton (1971). Let W denote the entrepreneur's liquid wealth process. The entrepreneur invests an amount of π_t in a diversified market portfolio and the remaining amount $W_t - \pi_t$ in the risk-free asset with a constant interest rate r. The return of the diversified market portfolio is denoted by R which satisfies

$$dR_t = \mu_M dt + \sigma_M dB_t, \quad (1)$$

where μ_M and $\sigma_M > 0$ are constants, and $\eta \equiv (\mu_M - r)/\sigma_M$ is the Sharpe ratio of the market portfolio.

We assume the project generates a stochastic revenue process $\{y_t : t \geq 0\}$ that follows a geometric Brownian motion (GBM):

$$dy_t = \mu_y y_t dt + \rho\sigma y_t dB_t + \epsilon y_t dZ_t, \quad y_0 \text{ given}, \quad (2)$$

where μ_y is the expected growth rate, σ is the total volatility and $\rho \in [-1, 1]$ is the correlation coefficient between the project payoff and the return on the market portfolio given by (1). A higher absolute value $|\rho|$ of the correlation coefficient implies that the systematic volatility has a larger weight, *ceteris paribus*. The parameters $\omega \equiv \rho\sigma$ and $\epsilon \equiv \sqrt{1-\rho^2}\sigma$ are respectively the systematic and idiosyncratic volatility of the revenue growth.

2.2. Entrepreneurial financing with equity-for-guarantee swap

We assume that the entrepreneur runs the project by setting up a limited liability entity, such as a limited liability company (LLC) or an S corporation, which allows her to face single-layer taxation for her business income and makes the debt nonrecourse. We follow the simple tax system in Chen et al. (2010). Entrepreneurial business profits incur taxes at a rate τ_e. A public firm is subject to a double taxation which is captured by an effective marginal tax rate τ_m. The capital gains upon cash-out are taxed at a rate τ_g.

The entrepreneur finances the initial one-time lump-sum cost I via her own funds and external financing. We assume that the main source of external financing is debt, e.g. bank loans. Due to the high default probability and relative lack of collateral, it is much more difficult for the entrepreneurial firm to take debt financing than for a large company. Unlike Chen et al. (2010) who do not consider borrowing constraints, we study the entrepreneur who is constrained in borrowing due to protected covenants demanded by the lenders. This financing constraint is alleviated by introducing the EGS supported by a commercial guarantee company or insurer. Unlike the traditional credit hypothecation, though, the entrepreneurial firm in the new credit guarantee scheme must pay to the guarantee company a portion (φ) of equity as guarantee costs instead of regular guarantee fees.

Under the guarantee, the entrepreneur chooses to issue an interest-only consol with coupon b and par value $F_0 = F(y_0)$ at time 0 and remains unchanged until the entrepreneur exits from the project, see (A.9) and (A.11). After the debt is in place, at any time $t \geq 0$, the entrepreneur has three choices: (1) She runs the firm and receives a fraction $(1 - \varphi)$ of cash payments from the firm; (2) She defaults once the default threshold y_d of the revenue process is reached and then the insurer must make a compensatory payment to the creditor so that the creditor is paid up-to a certain guarantee level; (3) She cashes out by selling the firm to a diversified buyer at the cash-out threshold y_u, which incurs a fixed transaction cost K.

Once the entrepreneur defaults, the debt holders (lenders) take control and liquidate/sell the firm. Bankruptcy ex post is costly and the bankruptcy loss can be interpreted in different ways, such as loss from selling real assets, asset fire-sale losses, legal fees, etc. We assume that $\kappa \equiv 1 - \alpha$ is the bankruptcy loss rate, i.e. α is the recovery rate. Then the remaining liquidation/sale value of the firm is equal to $\alpha A(y_d)$, where $A(y_d)$ is the equilibrium value of an unlevered (all-equity without debt) public firm given by (A.1). Moreover, the debt holders will gain the compensatory payment from the guarantee company so that under the arrangement of equity-for-guarantee contract the debt holders gain $\phi b/r$ once the entrepreneur defaults instead of the remaining value $\alpha A(y_d)$ only, where ϕ is the guarantee level.

[2] As argued by Miao and Wang (2007) and Chen et al. (2010), it is believed that our model and insights apply to regular utility functions, since the precautionary savings effect, which is captured by utility functions with convex marginal utility like CARA, is the driving force. For this reason and mathematical convenience, we only consider CARA utility in the paper.