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Innovative Applications of O.R.

A branch-and-price-and-cut approach for sustainable crop rotation planning

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ABSTRACT

In this paper, we study a multi-periodic production planning problem in agriculture. This problem belongs to the class of crop rotation planning problems, which have received considerable attention in the literature in recent years. Crop cultivation and fallow periods must be scheduled on land plots over a given time horizon so as to minimize the total surface area of land used, while satisfying crop demands every period. This problem is proven strongly *NP*-hard. We propose a 0-1 linear programming formulation based on crop-sequence graphs. An extended formulation is then provided with a polynomial-time pricing problem, and a Branch-and-Price-and-Cut (BPC) algorithm is presented with adapted branching rules and cutting planes. The numerical experiments on instances varying the number of crops, periods and plots show the effectiveness of the BPC for the extended formulation compared to solving the compact formulation, even though these two formulations have the same linear relaxation bound.

1. Introduction

Although definitions of sustainable agriculture may vary, agricultural systems are generally considered as sustainable if they sustain themselves along a long period of time, that is, if they are economically viable, environmentally safe, and socially fair. In particular, sustainable agricultural practices are usually requested to incorporate alternatives to toxic fertilizers and pesticides, avoid excessive tillage and preserve soils. Many research papers about sustainable agriculture focus on the pollution and social side-effects of intensive agriculture, such as water spoiled by pesticides, crop diseases, and concentration of production in fewer and bigger farms that can afford large investments in costly automative production systems and technologies (e.g., Altieri, 1995; Gliessman, 1998). Consistent with this diagnosis, recommendations on the need for new sustainable agricultural systems can be found in Tilman, Cassman, Matson, Naylor, and Polasky (2002). Crop rotations, combined with fallow periods where the land rests in order to recover its soil attributes after production, enable crop diversification on both space and time dimensions. Typical crop rotation problems usually focus on building rotations that maximize a profit or yield function, where the total surface area of land is either unbounded or fixed (Detlefsen & Jensen, 2007; El-Nazer & McCarl, 1986; Haneveld & Stegeman, 2005; dos Santos, Michelon, Arenales, & Santos, 2011). This paper deals with an aspect of sustainability which is rarely considered in optimization of agricultural production systems: the minimization of the surface area needed to cover crop demands that vary over time. A compact formulation for a mixed-integer variant of the problem was originally introduced in Alfandari, Lemalade, Nagih, and Plateau (2011), following a communication in the EURO XXI Conference in 2006. Since then, a number of papers have addressed crop rotation planning in a sustainable development context. For example, a column generation approach was applied in dos Santos et al. (2011), where the objective is to maximize space occupation and the master problem includes adjacency constraints between plots. Column generation was also used in dos Santos, Costa, Arenales, and Santos (2010) for a crop rotation problem with land divided into plots, but with continuous variables representing the surface area assigned to a given rotation, hence requiring no branching. Another example can be found in Costa, dos Santos, Alem, and Santos (2014) where harvested crops can be stocked for a limited period of time, and demands are subject to uncertainty. Rdulescu, Rdulescu, and Zbganu (2014) present multi-objective crop rotation models that take risk into account, converted into linear programs and solved with standard Linear Programming (LP) methods. A survey on crop rotation decisions exists (Dury, Schaller, Garcia, Reynaud, & Bergez, 2012) but does not include most recent papers in optimization. Dantzig–Wolfe decomposition (Dantzig & Wolfe, 1960) was applied to our problem

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in Sadki (2011), but with no inclusion in a branch-and-price approach to obtain optimal integer solutions. To our knowledge no branch-and-price algorithm has ever been designed so far for any crop rotation planning problem.

The original model presented in Alfandari et al. (2011) was motivated by a Madagascan case study where the minimization of cultivated space contributed to the sustainable development of the primary forest in the long term. Indeed, farmers in Madagascar are used to clearing more and more primary forest areas – although this is prevented by law – in order to extend their cultivation surface area to better cover their needs. A plot could be cultivated with several crops in the same period in this study. We direct the reader to Alfandari et al. (2011) for more details on the agricultural Madagascan context. In this paper, we present a fully-combinatorial problem where a single crop can be cultivated on each plot at each period.

The paper is organized as follows. Section 2 introduces notation and crop-sequence graphs. Section 3 describes a compact formulation of the problem. Section 4 proves *NP*-hardness. Section 5 provides a covering integer programming extended formulation derived from a Dantzig–Wolfe decomposition approach. Section 6 presents the branch-and-price-and-cut with branching rules and cutting planes. Section 7 presents computational experiments for various time horizons, number of crops and plot sizes. Section 8 concludes the paper.

2. Notations and crop-sequence graphs

We consider the following notations for the Minimum-Space Crop Rotation Planning problem (*MSCRP*):

- $t = 1, \ldots, T$: the periods of the planning horizon.
- $p = 1, \ldots, P$: the set of land plots that can possibly be used.
- C: the set of crops, with $f \in C$ the fallow index considered as a specific crop for modeling reasons.
- $C_t \subseteq C$: the set of crops that can be cultivated at period t.
- d_{ct}: the demand (in tons) of crop $c \in C_t \setminus \{f\}$ at period t.
- L: the number of fallow periods after which the yield no longer increases.
- L': the maximum number of consecutive periods a plot can be cultivated before returning fallow.
- s_p: the surface area of plot p (in ha).

The *state* of a plot p is a triplet $v = (c, l, l')$ where c is the crop (or fallow) at the current period, $l \leq L$ is the *fallow length*, i.e., the number of consecutive fallow periods before cultivation (if this number is greater than L then it is replaced by L), and $l' \leq L'$ is the *cultivation length*, i.e. the number of consecutive cultivation periods up to the current period. The only possible states are $(f, l, 0)$ for $l = 1, \ldots, L$, and (c, l, l') for $c \in C \setminus \{f\}$, $l = 1, \ldots, L$, $l' = 1, \ldots, L'$. When the plot is cultivated in period t and remains cultivated in the next period $t + 1$ the cultivation length l' of the plot is increased by one. When the maximum length of cultivation L' is reached, the plot has to return fallow, with fallow length $l = 1$ and cultivation length $l' = 0$. When a plot has been left fallow for l periods, it can either remain fallow the next period with length $\min\{l + 1, L\}$ or go back to cultivation with some crop c, fallow length l and cultivation length $l' = 1$. We denote by $Succ(v)$ the set of possible successors of state v at the next period, and by $Pred(v)$ the set of predecessors of state v at the previous period. Fig. 1 provides the list of possible successors and predecessors of each state if $C = \{rice, bean, f\}$, rice precedes bean and bean precedes rice.

State v	$Succ(v)$	$Pred(v)$
$(rice, l, 1)$	$(bean, l, 2)$, $(f, 1, 0)$	$(f, l, 0)$
$(rice, l, l')$, $1 < l' < L'$	$(bean, l, l' + 1)$, $(f, 1, 0)$	$(bean, l, l' - 1)$
$(rice, l, L')$	$(f, 1, 0)$	$(bean, l, L' - 1)$
$(f, l, 0)$	$(f, \min(l + 1, L), 0), (rice, l, 1), (bean, l, 1)$	$(rice, l, l'), (bean, l, l')$

Fig. 1. Table of successors and predecessors of a state with two alternating crops.

We denote by V_{pt} the set of possible states of plot p at period t. At the beginning of the planning horizon, V_{p0} is reduced to a single state $start_p$. We note for $t = 1, \ldots, T$

$$A_{pt} = \{(v, v') \in V_{p,t-1} \times V_{pt} \mid v' \in Succ(v)\}$$

the set of possible *transitions* from a state $v \in V_{p,t-1}$ to a state $v' \in V_{pt}$, as illustrated by Fig. 1. We also note, for each state $v \in V_{pt}$, $A^+_{pt}(v) = \{(v, v') \mid (v, v') \in A_{p,t+1}\}$ and $A^-_{pt}(v) = \{(v', v) \mid (v', v) \in A_{pt}\}$ the set of transitions that start at state v and end at state v at period t, respectively.

Now, consider the acyclic directed graph $G^p = (V^p, A^p)$ with $V^p = \cup_{0 \leq t \leq T} V_{pt} \cup \{end_p\}$, where node end_p represents the end of a rotation, and $A^p = \cup_{1 \leq t \leq T} A_{pt} \cup \{(v, end_p) \mid v \in V_{pT}\}$. We call this graph the *crop-sequence graph*. By construction, any path from $start_p$ to end_p in graph G^p identifies a feasible crop rotation on plot p. For each crop $c \in C_t \setminus \{f\}$, we call $A^c_{pt} \subset A_{pt}$ the set of arcs such that crop c is cultivated at the final endpoint of transition a. Each arc $a \in A^c_{pt}$ is valued by $s_p w_{pac}$, where w_{pac} is the number of tons of crop c obtained by transition a on one hectare of plot p. All other arcs, i.e., those which have a fallow state $(f, l, 0)$ as final endpoint and those that terminate at end_p, have a zero value.

Fig. 2 describes such a crop-sequence graph for two possible crops rice (r) and bean (b) and five periods. The following section describes a compact formulation of the problem.

3. Compact formulation

MSCRP problem is that of constructing crop rotations minimizing the total space area required for covering seasonal crop demands. We introduce the following Compact Formulation (*CF*) for *MSCRP*.

$$\min \quad \sum_{p=1}^{P} \sum_{a \in A_{p1}} s_p x_{pa1} \tag{1}$$

$$\text{s.t} \quad \sum_{p=1}^{P} \sum_{a \in A^c_{pt}} s_p w_{pac} x_{pat} \geq d_{ct} \qquad \forall c \in C_t \setminus \{f\}, t = 1, \ldots, T \tag{2}$$

$$(CF) \quad \sum_{a \in A^-_{pt}(v)} x_{pat} = \sum_{a \in A^+_{pt}(v)} x_{pa,t+1} \qquad \forall p = 1, \ldots, P, \; t = 1, \ldots, T-1, v \in V_{pt} \tag{3}$$

$$x_{pat} \in \{0, 1\} \tag{4}$$

Binary decision variable x_{pat} is equal to one if and only if plot p uses transition a in period t. The objective function (1) minimizes the total surface area of plots p that are used for production, i.e. such that $\sum_{a \in A_{p1}} x_{pa1} = 1$. Global constraints (2) ensure that the total production of a crop is at least the demand for every period. Flow conservation constraints (3) are local constraints associated with a plot p and define a path structure for a rotation on that plot. Note that if $\sum_{a \in A_{p1}} x_{pa1} = 0$, for this plot p all variables x_{pat} are equal to zero which means that no crop rotation is used on that plot. The linear relaxation of *CF* will be noted $\overline{CF}$. We study the complexity of the *MSCRP* problem in the following section.

4. Problem complexity

We prove the *NP*-hardness of this problem with a polynomial reduction from the (unweighted) Set Covering Problem (*SCP*). In the unweighted *SCP*, we are given a set of elements $I = \{1, \ldots, n\}$ and a collection of subsets of elements $\mathcal{S} = \{S_1, \ldots, S_m\}$ such that $S_j \subset I$. The objective is to find a collection $\mathcal{S}' \subset \mathcal{S}$ such that $\bigcup_{S_j \in \mathcal{S}'} S_j = I$ and the size $|\mathcal{S}'|$ of the cover is minimum.

Theorem 1. *MSCRP is strongly NP-hard and reduces to the Set Covering Problem.*

Proof. We transform a general *SCP* instance into a specific *MSCRP* instance in the following way. Take $P = m$, $T = m + n$, $C = \{f, 0\} \cup I =$