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Innovative Application of O.R.

A column generation approach for a multi-attribute vehicle routing problem

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ABSTRACT

In this paper, we consider a multi-attribute vehicle routing problem derived from a real-life milk collection system. This problem is characterized by the presence of a heterogeneous fleet of vehicles, multiple depots, and several resource constraints. A branch-and-price methodology is proposed to tackle the problem. In this methodology, different branching strategies, adapted to the special structure of the problem, are implemented and compared. The computational results show that the branch-and-price algorithm performs well in terms of solution quality and computational efficiency.

1. Introduction

The vehicle routing problem (VRP) lies at the center of logistics and distribution management and is one of the most studied problems in the field of operations research. Numerous variants have been studied since the problem was first introduced by Dantzig and Ramser (1959). The simplest problem in this domain is the capacitated vehicle routing problem (CVRP). In the CVRP, all the customers correspond to deliveries. The customers' demands are deterministic, known in advance, and may not be split. The vehicles are identical and based at a single central depot. Each vehicle can perform only one route, and the quantity supplied cannot exceed the vehicle capacity. The objective most commonly used is to minimize the total cost (i.e., a weighted function of the number of routes and their length or travel time) of serving all the customers (Toth & Vigo, 2002).

Many real-world combinatorial optimization problems, including logistics applications and transportation problems, have several complicating attributes. These attributes lead to the characteristics, constraints, and objectives that define the problem. When there are many attributes the problem becomes complex and challenging. In the combinatorial optimization literature, such problems are called "multi-attribute problems." Recently, the research community has focused on simultaneously considering multiple attributes, to provide more representative models of real-world situations. In particular, VRP researchers have recently concentrated on multi-attribute vehicle routing problems (MAVRP; see Hartl, Hasle, & Janssens, 2006). They have explored several variations of the MAVRP, each representing a specialized extension of the classical VRP and reflecting a real-world application. However, not all variants have received the same attention. Furthermore, most of the contributions have developed heuristics and metaheuristics, and there are few efficient exact algorithms for the variants of the MAVRP.

We introduce a new MAVRP variant that incorporates some common real-world features. It is inspired by collection-redistribution activities in the raw-milk industry of Quebec. This problem consists of route planning for a heterogeneous fleet of vehicles departing from different depots. The vehicles must visit a set of producers in specific time windows, and the collected product is then delivered to processing plants. Finally, the vehicles return to their home depots. The most similar model in the literature is the multi-depot heterogeneous vehicle routing problem with time windows (MDHVRPTW).

The main goal of this paper is to investigate the challenges of complex problems with features such as collection-redistribution activities. We formulate a multi-attribute VRP with certain special features that takes the form of an MDHVRPTW with deliveries to plants. The main contributions of this paper are summarized as follows:

- We introduce a variant of the MAVRP. It differs from well-studied variants such as VRPTW and MDVRPTW because there is an

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extra level of difficulty associated with the assignment of routes to plants.
- We propose a set partitioning formulation for this problem.
- We develop a branch-and-price algorithm. It includes a number of structural exploration and exploitation features that improve the computational efficiency of the solution strategy.
- We perform an extensive analysis using a large set of randomly generated instances, to illustrate the efficiency of the algorithm and investigate the characteristics of the problem.

The remainder of the paper is organized as follows. In Section 2, we describe in detail the problem class and its different variants. In Section 3, we give a brief literature review to better position the present study. In Section 4, we choose a special case of the problem class and present the set partitioning model. In Section 5, we present the proposed solution methodology, and experimental results are given in Section 6. Finally, Section 7 provides concluding remarks.

2. Problem class

In this section, we introduce a new MAVRP variant inspired by the dairy transportation problem in Quebec (see Lahrichi, Crainic, Gendreau, Rei, & Rousseau, 2013). Basically, it consists of constructing collection routes that are then assigned to plants that receive the collected products. It is usually encountered in the collection and redistribution of perishable products. There are three types of stakeholders, as described below:

- The producers, which periodically produce a limited quantity of one or more products.
- The plants, which periodically receive the products. They transform these raw materials into consumable goods.
- The carriers, which collect the products from the producers and deliver them to the plants. Each carrier has one or more depots where the vehicles are located.

The vehicles usually have different capacities, fixed costs, and variable costs. The fixed costs are the expenses that are not related to the distance traveled and have to be paid when the vehicle is used; the variable costs depend on the distance traveled. In most applications, each producer has an associated time window indicating the earliest and latest collection times. Each plant has an associated demand window indicating the minimum and maximum quantities that can be delivered.

A route is a path that starts and ends at a depot and visits producers and plants; it may contain one or more pick-up and delivery phases. A route is feasible if the pick-ups do not exceed the vehicle capacity and the associated time windows are respected. The cost of a route is the sum of the costs of the arcs on the path plus the sum of the vehicle's fixed and variable costs. We assume throughout this paper that the triangle inequality holds for the costs and travel times. Also, the service times are considered to be independent of the quantities collected or delivered.

There may be some preassignments based on contractual restrictions, strategic/tactical planning decisions, or equipment compatibility. We introduce three: (1) producer-depot preassignments, which assign a producer to a specific depot; (2) producer-plant preassignments, which specify which plant receives the products of a given producer; (3) producer-depot-plant preassignments, which assign a producer to a depot and a plant. The most general variant of the problem has no preassignments.

A vehicle can perform one or more circuits per day. We define three route types as follows:

Simple route: Each vehicle visits several producers and collects their products. It then delivers its entire load to one plant and returns to its depot (Fig. 1a).

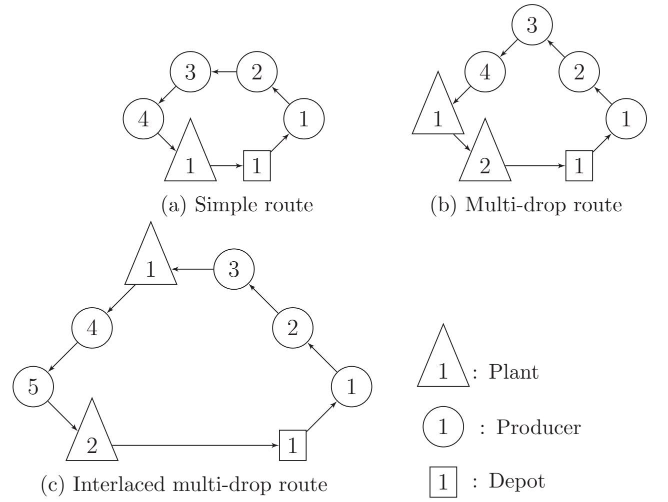

Fig. 1. General route configurations.

Multi-drop route: A vehicle delivers its load to more than one plant before returning to its depot (Fig. 1b).

Interlaced multi-drop route: Vehicles perform several circuits per day. A vehicle may visit other producers after completing its first visit to a plant. One or more plants are visited (Fig. 1c).

We consider simple routes, and Section 4 gives the details of this subclass of the problem.

3. Literature review

In this section, we review research into different variants of the MAVRP. We focus on exact algorithms rather than heuristic methods and consider variants of the VRP with attributes similar to those of our problem.

Among the variants of the VRP, the VRPTW has received the most attention, and numerous researchers have applied column generation methodology to solve the problem. For the VRPTW, column generation was first used by Desrochers, Desrosiers, and Solomon (1992) in a Dantzig–Wolfe decomposition framework. They devised a branch-and-bound algorithm to solve a number of original time-window constrained problems from Solomon (1986) to optimality or near optimality. Kohl, Desrosiers, Madsen, Solomon, and Soumis (1999) improved the method by adding 2-path inequalities to the LP relaxation of the set partitioning formulation. Kohl and Madsen (1997) proposed a branch-and-bound algorithm in which subgradient and bundle methods were employed to compute the lower bounds. These methods were based on 2-cycle elimination algorithms. Irnich and Villeneuve (2006) proposed a branch-and-price algorithm in which the subproblem is solved using a k-cycle elimination procedure. Branch-and-price has been the leading methodology for the VRPTW since the beginning of the 1990s.

Feillet, Dejax, Gendreau, and Gueguen (2004) improved the extension of the Ford–Bellman algorithm proposed by Desrochers (1988). More precisely, they improved the labeling procedure for the elementary shortest path problem with resource constraints (ESPPRC), which is the backbone of a number of solution procedures based on column generation, by proposing new labels and dominance rules. Righini and Salani (2004) proposed an improved bounded bidirectional label-correcting algorithm in which two sequential labeling processes starting from the depot and a copy of the depot (considered the sink node) cooperate to accelerate the solution of the ESPPRC.

The most efficient algorithms for the ESPPRC are based on a partial or complete relaxation of the elementarity condition. Boland, Dethridge, and Dumitrescu (2006) and Righini and Salani (2009)