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Short Communication

Two faster algorithms for coordination of production and batch delivery: A note

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ABSTRACT

This note suggests faster algorithms for two integrated production/distribution problems studied earlier, improving their complexities from $O(n^{2V+4})$ and $O(n^2(L+V)^2)$ to $O(n)$ and $O(n+V\min\{V,n\})$ respectively, where n is the number of products to be delivered, V is the number of vehicles and L is the number of vehicle departure times.

1. Introduction

Agnetis, Aloulou, and Fu (2014) provided complexity results and solution algorithms for various problems and models concerning the coordination of production and inbound delivery in a supply chain. In particular, they have addressed two problems we are going to describe hereafter.

A few features are common to both problems. There is a set $N=\{1,\dots,n\}$ of n products which are manufactured according to a fixed (given) schedule. The completion time C_j of any product j is given, $j=1,\dots,n$. This is therefore the time at which a product j is released by the manufacturing (upstream) stage for delivery. We index the jobs so that $C_1\le\dots\le C_n$. The time at which a vehicle reaches the destination (downstream stage) is called *delivery time*. Dealing with inbound distribution, any product must be delivered to the downstream stage of the supply chain within T time units since its release by the manufacturing stage (*responsiveness constraint*). The delivery of the products is performed by V vehicles, each of which can carry up to c, $c<n$, products at once. The set of products assigned to the same vehicle in one delivery is called a *batch*. The objective is to deliver the products complying with the responsiveness constraint and using the minimum number of batches.

The two problems differ in the *transportation mode*. In the first problem, there are no constraints on the starting time of a vehicle, i.e., if a vehicle is present at the upstream stage, it can start at any time. A vehicle takes time l to reach the downstream stage and another l' time units to go back to the upstream stage. After returning, a vehicle is again available for delivering another batch. In this case, we say that vehicles are *express* and we denote the problem as *P-express*.

(***P*-express**). *Given a set $N=\{1,\dots,n\}$ of n products, a fixed release time C_j for each product j, $j=1,\dots,n$, V express vehicles, a travel time l from the upstream to the downstream stage, a travel time l' to go back to the upstream stage and a responsiveness parameter T, find a partition of N into batches and an assignment of batches to vehicles so that each product is delivered to the downstream stage within C_j+T, $j=1,\dots,n$, and the number of batches is minimum.*

In the second problem, there are L *fixed* delivery departure times $t_1<\dots<t_L$. At time t_i, $i=1,\dots,L$, there are $v_i\le n$ vehicles ready to leave. Each vehicle takes time l to reach the downstream stage and does not make return, hence in this problem $V=\sum_{i=1}^{L}v_i$. In this case, we denote the problem as *P-regular*.

(***P*-regular**). *Given a set $N=\{1,\dots,n\}$ of n products, a fixed release time C_j for each product j, $j=1,\dots,n$, a set $t_1,\dots,t_L$ of L departure times, a number v_i of vehicles ready to start at time t_i, $i=1,\dots,L$, a forward travel time l and a responsiveness parameter T, find a partition of N into batches and an assignment of batches to vehicles so that each product is delivered to the downstream stage within C_j+T, $j=1,\dots,n$, and the number of batches is minimum.*

Agnetis et al. (2014) addressed a more general problem, in which both express and regular transportation modes can be used for product delivery. Problems *P-express* and *P-regular* are therefore special cases of this problem. For these two special cases, Agnetis et al. (2014) presented algorithms with running times $O(n^{2V+4})$ and $O(n^2(L+V)^2)$,

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respectively. In this note (Sections 2 and 3), we demonstrate that these problems can be solved in $O(n)$ and $O(n + V\min\{V, n\})$ time, respectively.

2. Express vehicles

In this section, we reduce problem *P-express* to a known batch scheduling problem. This reduction allows deriving an $O(n)$ time algorithm, which improves the previous complexity result of $O(n^{2V+4})$ by Agnetis et al. (2014), Section 4.3.

We denote the following batch scheduling problem as *P-batch*. There are $\tilde{n}$ jobs and m parallel identical machines. All jobs have the same processing time $p_j = p, j = 1, \ldots, \tilde{n}$. Each job has a release date $r_j \geq 0$ and a deadline d_j, $d_j \geq r_j + p, j = 1, \ldots, \tilde{n}$. Jobs are performed in batches having bounded capacity q, i.e., a machine can handle up to q jobs in parallel. Jobs in a batch have the same starting and completion time (this is also known as *batch availability model with parallel job processing and bounded batch capacity*, see Potts and Kovalyov (2000) for the terminology). Since all job processing times are identical and equal to p, the processing time of any batch equals p. The problem is to find a feasible batch schedule (i.e., a partition of $\tilde{N} = \{1, \ldots, \tilde{n}\}$ into batches and an assignment of batches to machines that respect all release dates and deadlines) with the minimum number of batches.

(***P*-batch**). *Given a set $\tilde{N} = \{1, \ldots, \tilde{n}\}$ of $\tilde{n}$ jobs, each having a processing time $p_j = p$, a release date $r_j \geq 0$ and a deadline d_j, $j = 1, \ldots, \tilde{n}$, a set of m parallel identical batching machines of capacity q, find a feasible batch schedule so that the number of batches is minimum.*

In what follows, we show that *P-express* can be reduced to *P-batch*. In such reduction, express vehicles in *P-express* correspond to machines in *P-batch*, products in *P-express* to jobs in *P-batch* and the vehicle round trip, including batch delivery and return in *P-express*, to the processing of the corresponding batch in *P-batch*. More formally, given an instance I of *P-express*, construct an instance I_B of the problem *P-batch* as follows:

$$\begin{aligned} &\tilde{n} := n; \\ &m := V; \\ &q := c; \\ &r_j := C_j, \quad d_j := r_j + T + l', \quad p_j = p = l + l', \quad j = 1, \ldots, \tilde{n}. \end{aligned} \tag{1}$$

Lemma 1. *Given a feasible solution for the instance I_B of problem P-batch, a feasible solution for the instance I of problem P-express can be built having the same value of objective function, and vice versa.*

Proof. Consider an arbitrary batch G in a feasible solution of I_B. Let $\tilde{C}_G$ denote the completion time of this batch. Due to the feasibility, the relation $\max_{j\in G}\{r_j\} + p \leq \tilde{C}_G \leq \min_{j\in G}\{d_j\}$ is satisfied. This relation can be written as

$$\max_{j\in G}\{C_j\} + l + l' \leq \tilde{C}_G \leq \min_{j\in G}\{C_j\} + T + l',$$

which implies $\max_{j\in G}\{C_j\} \leq \tilde{C}_G - l' - l$ and $\tilde{C}_G - l' \leq \min_{j\in G}\{C_j\} + T$. Now, associate with batch G a batch H in the corresponding instance I of *P-express*, and let $\tilde{C}_G - l' - l$ and $\tilde{C}_G - l'$ be its start and delivery times, respectively. The latter two relations prove that the products in the batch H are feasibly delivered for the problem *P-express*. Also note that, according to reduction (1), no two batches in I will be assigned to the same vehicle at the same time and each vehicle returns empty to the upstream stage.

Conversely, let H be a batch in a feasible solution of I. The corresponding vehicle departs not earlier than $\max_{j\in H}\{C_j\}$ and delivers not earlier than $\max_{j\in H}\{C_j\} + l$. Let $\tau_{DEL,H}$ be the delivery time of batch H in the feasible solution to I. Due to feasibility, one must have

$$\tau_{DEL,H} - \min_{j\in H}\{C_j\} \leq T. \tag{2}$$

Now, define the completion time of the corresponding batch G in I_B as $\tilde{C}_G := \tau_{DEL,H} + l'$. Then, from (2), we obtain

$$\tilde{C}_G \leq \min_{j\in H}\{C_j\} + T + l' = \min_{j\in G}\{d_j\}.$$

Hence, all job deadlines in I_B are satisfied. Furthermore, batch G in I_B starts at time $\tilde{C}_G - p$, and

$$\tau_{DEL,H} + l' - p = \tau_{DEL,H} - l \geq \max_{j\in H}\{C_j\} = \max_{j\in G}\{r_j\},$$

so batch G respects job release dates as well. Finally, each vehicle carries one batch and hence no machine will process more than one batch at a time. □

We are now in the position of proving the following result.

Theorem 1. *Problem P-express can be solved in $O(n)$ time.*

Proof. Given an instance of *P-express*, it can be solved by constructing the corresponding instance of *P-batch* according to (1) and solving it. Koehler and Khuller (2013) developed an $O(\tilde{n})$ time algorithm for *P-batch* in the special case in which release dates and deadlines are *agreeable*, i.e., $r_i < r_j$ implies $d_i \leq d_j$ for any jobs i and j. We observe that, due to the reduction (1), the instance of *P-batch* obtained is indeed agreeable, and as a consequence, the instance of *P-express* can solved in $O(n)$ time. □

3. Regular vehicles

In this section, we study the problem *P-regular* and develop an $O(n + V\min\{V, n\})$ time algorithm for it, which improves the previous complexity result of $O(n^2(L + V)^2)$ by Agnetis et al. (2014).

In what follows, given two integers A and B, $B \geq A$, we let $[A, B]$ denote the closed interval of integers $A, A + 1, \ldots, B$. Recall that $C_1 \leq \cdots \leq C_n$ and $t_1 < \cdots < t_L$.

Given an instance of *P-regular*, consider the ith departure time t_i. Denote by $[A_i, B_i] \subseteq N$ the maximal interval of products such that any product from this interval can be feasibly delivered if its delivery starts at time t_i, $i = 1, \ldots, L$. Observe that product j can be feasibly delivered by a vehicle departing at time t_i if and only if $t_i - T + l \leq C_j \leq t_i$, $j = 1, \ldots, n$. All the intervals $[A_i, B_i]$, $i = 1, \ldots, L$, can be constructed in $O(n + L)$ time.

The following two lemmas are trivial.

Lemma 2. *$A_1 \leq \cdots \leq A_L$ and $B_1 \leq \cdots \leq B_L$.*

Lemma 3. *Problem P-regular has a solution only if $\cup_{i=1}^{L}[A_i, B_i] = N$.*

Note that if an instance of *P-regular* has a solution, then $A_1 = 1$, $B_L = n$ and $A_{i+1} \leq B_i$, $i = 1, \ldots, n - 1$. Moreover, a simple product interchange argument can be used to prove the following:

Lemma 4. *If the problem P-regular has a solution, then there exists an optimal solution, in which each batch consists of consecutively indexed products and a batch with smaller product indices is delivered earlier.*

We will consider solutions which satisfy Lemma 4. In what follows, we conveniently represent an instance and a feasible solution by means of the diagram shown in Fig. 1. There are L rows and n columns. Row i corresponds to departure time t_i, and circles in these row correspond to the products that can be feasibly delivered by a vehicle departing at t_i. Note that Lemma 4 implies that, if a feasible solution exists, no column can be empty. We represent a feasible solution by framing the products in the same batch. Products assigned to a batch are marked in bold. For the example in Fig. 1, there are $L = 4$ delivery times, batch sizes are bounded by $c = 4$, and the numbers of vehicles are $v_1 = 4$, $v_2 = 3$, $v_3 = 3$ and $v_4 = 2$.

We call product $j \in [A_i, B_i]$ a *product in row i*. Note that a product is in at least one row (because of Lemma 3) and, in general, is in several rows. Given a feasible solution, we call a batch *full* if it contains exactly