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Short Communication

Parameter estimation based on interval-valued belief structures

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ABSTRACT

Parameter estimation based on uncertain data represented as belief structures is one of the latest problems in the Dempster–Shafer theory. In this paper, a novel method is proposed for the parameter estimation in the case where belief structures are uncertain and represented as interval-valued belief structures. Within our proposed method, the maximization of likelihood criterion and minimization of estimated parameter's uncertainty are taken into consideration simultaneously. As an illustration, the proposed method is employed to estimate parameters for deterministic and uncertain belief structures, which demonstrates its effectiveness and versatility.

1. Introduction

Dempster–Shafer theory (D–S theory for short) (Dempster, 1967; Shafer, 1976) has been widely used because it allows to handle uncertain data (Durbach & Stewart, 2012; Yang, Yang, Liu, & Li, 2013; Yang & Xu, 2013). In D–S theory, various belief structures are employed to represent the uncertain data. Recently, the study of parameter estimation based on belief structures has attracted many attentions (Come, Oukhellou, Denoeux, & Aknin, 2009; Denoeux, 2010, 2013; Su, Wang, & Wang, 2013). Typically, Denoeux (Denoeux, 2013) proposed an evidential EM algorithm for parameter estimation in the case of crisp belief structures, and Su et al. (2013) developed a parameter estimation approach for fuzzy belief structures. In this paper, the parameter estimation based on interval-valued belief structures (Wang, Yang, Xu, & Chin, 2006; Yager, 2001) has been considered. A novel parameter estimation method is proposed for the case of interval-valued belief structures. Within the proposed method, two criteria, the maximization of observation data's likelihood and the minimization of estimated parameter's uncertainty, are both considered simultaneously. The proposed method is effective for both crisp (deterministic) and interval-valued (uncertain) belief structures, and promising for various applications.

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2. D–S theory and belief structures

D–S theory (Dempster, 1967; Shafer, 1976) is often regarded as an extension of the Bayesian theory. Please refer to Shafer (1976) and Yang and Xu (2013) for more knowledge about D–S theory. In D–S theory, various belief structures, such as crisp, interval-valued and fuzzy belief structures, are employed as basic data structures. They are used to express various uncertain information. A crisp belief structure is defined as follows.

Definition 1. Let a finite nonempty set Ω be a frame of discernment, and 2^{Ω} denote the power set of Ω. A crisp belief structure is a mapping $m : 2^{\Omega} \to [0, 1]$, satisfying

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^{\Omega}} m(A) = 1 \tag{1}$$

The crisp belief structure is deterministic because its belief degree is expressed by real numbers. By contrast, the interval-valued belief structure (IBS) is a kind of uncertain belief structures, which is an extension of the crisp belief structure. It is more capable to represent the uncertain information. Some basic concepts about IBS are given as below (Wang et al., 2006; Yager, 2001).

Definition 2. Let Ω be a frame of discernment, $F_1, F_2, \ldots, F_n$ be the n focal elements on Ω. An IBS m_I satisfies such conditions

1. $a_i \le m_I(F_i) \le b_i$, where $a_i, b_i \in [0, 1]$ and $i = 1, 2, \ldots, n$;

2. $\sum_{i=1}^{n} a_i \le 1$ and $\sum_{i=1}^{n} b_i \ge 1$;
3. $m_I(F) = 0, \forall F \notin \{F_1, F_2, \ldots, F_n\}$.

An IBS is valid if it satisfies $\sum_{i=1}^{n} a_i \le 1$ and $\sum_{i=1}^{n} b_i \ge 1$. In the rest of this paper, all the IBSs are valid.

3. Proposed parameter estimation method

In previous literatures (Denoeux, 2013; Su et al., 2013), parameter estimation based on crisp and fuzzy belief structures has been studied. However, the parameter estimation based on interval-valued belief structures is still an unsettled problem. In this paper, a novel parameter estimation method based on IBSs is proposed to fill that gap. Without loss of generality, some concepts about interval probabilities are introduced first.

3.1. Interval probabilities

Definition 3 (Guo & Tanaka, 2010)**.** Let X be a finite set $X = \{x_1, \ldots, x_n\}$, a set of intervals $P_I = \{I_i = [w_i^-, w_i^+], i = 1, \ldots, n\}$ satisfying $0 \le w_i^- \le w_i^+ \le 1$ is an interval probabilities of X if there are $w_i^* \in [w_i^-, w_i^+]$ for $i = 1, \ldots, n$ such that $\sum_{i=1}^{n} w_i^* = 1$.

Interval probabilities are the extension of point-valued probability mass functions, which can be degenerated to the classical probability distribution.

Definition 4 (Guo & Tanaka, 2010)**.** Let $P_I = \{I_i = [w_i^-, w_i^+], i = 1, \ldots, n\}$ be an interval probabilities, the αth ignorance of P_I, denoted as $I^\alpha(P_I)$, is

$$I^\alpha(P_I) = \frac{\sum_{i=1}^{n} (w_i^+ - w_i^-)^\alpha}{n} \tag{2}$$

Obviously, $I^\alpha(P_I) \in [0, 1]$. $I^\alpha(P_I) = 1$ for $I_1 = I_2 = \cdots = I_n = [0, 1]$ and $I^\alpha(P_I) = 0$ for the point-valued probabilities. $I^1(P_I)$ can be seen as an effective index to measure the uncertainty/imprecision of interval probabilities.

3.2. Likelihood function model for IBS

To do the parameter estimation under IBS environment, the likelihood function model for IBS should be developed first. Let X be a discrete random variable taking values in $\Omega_X = \{H_1, H_2, \ldots, H_q\}$, with interval probabilities $p_X(\cdot; \theta)$ which depends on unknown parameter $\Theta = \{\theta_i = [\theta_i^-, \theta_i^+], i = 1, \ldots, q\}$. There are several types of observational data.

If the observational data is completely certain, for example H_i happened, the likelihood function given a singleton H_i can be represented as

$$L(H_i; \Theta) = [\theta_i^-, \quad \theta_i^+] \tag{3}$$

If an event F, $F \subseteq \Omega_X$, is observed, the likelihood function given a subset F is now

$$L(F; \Theta) = [L_F^-, \quad L_F^+] \tag{4}$$

where

$$L_F^- = \max\left[\sum_{H_i \subseteq F} \theta_i^-, \quad \left(1 - \sum_{H_i \not\subseteq F} \theta_i^+\right)\right],$$

$$L_F^+ = \min\left[\sum_{H_i \subseteq F} \theta_i^+, \quad \left(1 - \sum_{H_i \not\subseteq F} \theta_i^-\right)\right]$$

If the observational data is described by a piece of uncertain belief structure — an IBS m_I, the likelihood function given such uncertain data is

$$L(m_I; \Theta) = [L_{m_I}^-, \quad L_{m_I}^+] \tag{5}$$

where

$$\begin{aligned} L_{m_I}^- / L_{m_I}^+ &= \min / \max \quad \sum_{i=1}^{n} m_I(F_i) L_{F_i}^* \\ s.t. \quad & \sum_{i=1}^{n} m_I(F_i) = 1 \\ & a_i \le m_I(F_i) \le b_i, \quad \forall i = 1, \ldots, n \\ & L_{F_i}^- \le L_{F_i}^* \le L_{F_i}^+, \quad \forall i = 1, \ldots, n \end{aligned} \tag{6}$$

Now assuming there are p observational data, expressed by p IBSs, $\boldsymbol{m_I} = (m_{I_1}, m_{I_2}, \ldots, m_{I_p})$. The likelihood of $\boldsymbol{m_I}$ is represented as

$$L(\boldsymbol{m_I}; \Theta) = [L_{\boldsymbol{m_I}}^-, \quad L_{\boldsymbol{m_I}}^+] = \left[\prod_{i=1}^{p} L_{m_{I_i}}^-, \quad \prod_{i=1}^{p} L_{m_{I_i}}^+\right] \tag{7}$$

3.3. Solution for parameter estimation

The likelihood function model developed above is the foundation for the parameter estimation based on IBSs. Depending on that, an optimization model P is proposed to make an estimation for parameter Θ.

$$\textit{Model } P: \quad \arg\max_{\Theta} \quad D\left(L(\boldsymbol{m_I}; \Theta), \ [0, 0]\right) - I^\alpha(\Theta) \tag{8}$$

where $I^\alpha(\Theta)$ is the αth ignorance of Θ, and $D(L(\boldsymbol{m_I}; \Theta), \ [0, 0])$ is a distance measure for two intervals $L(\boldsymbol{m_I}; \Theta)$ and $[0, 0]$ presented in

Table 1
Observational data represented as crisp belief structures.

Observation	1	2	3	4	5	6
$m(\{a\})$	1.0	1.0	1.0	0.3	0.0	0.0
$m(\{b\})$	0.0	0.0	0.0	0.3	1.0	1.0
$m(\{a, b\})$	0.0	0.0	0.0	0.4	0.0	0.0

Table 2
Results of parameter estimation for the case of crisp belief structures.

Probability	$p(a)$	$p(b)$
Denoeux's method (Denoeux, 2013)	0.6	0.4
Proposed method ($\alpha = 1$)	0.6	0.4

Table 3
Observational data represented as IBSs.

Observation	1	2	3	4
$m_I(\{H_1\})$	[0.30, 0.40]	[0.35, 0.45]	[0.10, 0.25]	[0.30, 0.45]
$m_I(\{H_2\})$	[0.10, 0.25]	[0.10, 0.20]	[0.30, 0.45]	[0.30, 0.50]
$m_I(\{H_3\})$	[0.25, 0.35]	[0.20, 0.30]	[0.35, 0.50]	[0.15, 0.40]
$m_I(\{H_1, H_2, H_3\})$	[0.10, 0.20]	[0.05, 0.15]	[0.10, 0.25]	[0.00, 0.20]

Table 4
Results of parameter estimation for the case of IBSs.

α's value	$P_I(H_1)$	$P_I(H_2)$	$P_I(H_3)$	$I^1(P_I)$
$\alpha = 1$	[0.9823, 0.9823]	[0.0000, 0.0000]	[0.0177, 0.0177]	0.0000
$\alpha = 2$	[0.8397, 0.9433]	[0.0057, 0.1093]	[0.0510, 0.1547]	0.1036
$\alpha = 3$	[0.5821, 0.9331]	[0.0122, 0.3632]	[0.0547, 0.4058]	0.3510
$\alpha = 4$	[0.4614, 0.9569]	[0.0085, 0.5040]	[0.0346, 0.5301]	0.4955
$\alpha = 5$	[0.2963, 0.8751]	[0.0324, 0.6112]	[0.0925, 0.6713]	0.5788
$\alpha = 6$	[0.2580, 0.8907]	[0.0288, 0.6615]	[0.0805, 0.7132]	0.6327
$\alpha = 7$	[0.3228, 0.9988]	[0.0002, 0.6763]	[0.0010, 0.6770]	0.6760
$\alpha = 8$	[0.2687, 0.9744]	[0.0055, 0.7112]	[0.0201, 0.7259]	0.7057
$\alpha = 9$	[0.1876, 0.9136]	[0.0235, 0.7496]	[0.0629, 0.7889]	0.7260
$\alpha = 10$	[0.2272, 0.9768]	[0.0050, 0.7546]	[0.0182, 0.7678]	0.7496
$\alpha = 20$	[0.1339, 0.9815]	[0.0042, 0.8518]	[0.0143, 0.8619]	0.8476