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Interfaces with Other Disciplines

How to properly decompose economic efficiency using technical and allocative criteria with non-homothetic DEA technologies

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ABSTRACT

We discuss how to properly decompose economic efficiency when the underlying technology is non-homothetic using alternative allocative and technical efficiency criteria. We first show that only under the production of one output and assuming the particular case of constant returns to scale homotheticity, we may claim that the standard radial models correctly measure pure technical efficiency. Otherwise, when non-homotheticity is assumed, we then show that these traditional estimations would measure an undetermined mix of technical and allocative efficiency. To restore a consistent measure of technical efficiency in the non-homothetic case we introduce a new methodology that takes as reference for the economic efficiency decomposition the preservation of the allocative efficiency of firms producing in the interior of the technology. This builds upon the so-called reversed approach recently introduced by Bogetoft et al. (2006) that allows estimating allocative efficiency without presuming that technical efficiency has been already accomplished. We illustrate our methodology within the Data Envelopment Analysis framework adopting the most simple non-homothetic BCC model and a numerical application. In this application we show that there are significant differences in the allocative and technical efficiency scores depending on the approach.

1. Introduction

Economic (overall) efficiency measurement based on the approach initiated by Farrell (1957) has received great attention from academics and practitioners. Since Farrell, economic efficiency originates from two different sources, viz. technical efficiency and allocative efficiency. In the spirit of his renowned decomposition, technical efficiency is estimated in first place as some measure of the gains obtained from moving the evaluated firm to the frontier of the production possibility set. The main argument behind this approach is that the measurement of allocative efficiency presumes technical efficiency since only on the production isoquant the rate of substitution between production inputs is well-defined and comparable with the ratio of market prices. Therefore, under the Farrell's approach, the analysis focuses on the isoquant corresponding to the observed output before estimating allocative efficiency. Specifically, Farrell (1957) resorted to radial movements in order to measure technical efficiency, relating this particular component to both the coefficient of resource utilization of Debreu (1951) and the inverse of the Shephard's distance function (Shephard, 1953). Indeed, and thanks to duality results (Shephard, 1953), allocative efficiency can be derived as a residual between the overall economic efficiency and its technical efficiency component As a result of this residual nature of the allocative efficiency term, where its technical efficiency counterpart is the driving component, the former has received much less attention in the literature. While there are many ways to define and calculate technical efficiency (oriented and non-oriented models, radial, additive, directional-based measures, etc.), the allocative efficiency problem of the firm in relation to the overall economic efficiency has been neglected.

However, this is changing nowadays. In contrast to Farrell's approach, Bogetoft, Färe, and Obel (2006) introduced a new method for estimating the potential gains from improving allocative efficiency without presuming that technical efficiency has already been accomplished. In particular, they propose to use a 'reversed' Farrell approach, first correcting for allocative efficiency and next for technical efficiency and, consequently, changing the traditional order to decompose overall efficiency. The rationale is that when a firm is inefficient, both the input and output orientations are feasible choices to gain efficiency, and allocative efficiency can be evaluated in alternative input or output isoquants. Following this thread, we show that approaching the problem of

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decomposing overall economic efficiency dealing with allocative efficiency in the first place sheds new light on the analysis of the economic behavior of inefficient firms, which must be taken into account by researchers.

Particularly, the fact that firms may solve technical inefficiencies by either reducing inputs or expanding outputs has relevant implications in empirical research, as these two alternative dimensions normally used by researchers when measuring technical efficiency—output or input—pass on to the concept of allocative efficiency. This is indeed central in our analysis since an inefficient firm situating inside the technology may have used inputs in excess for the observed level of output (input perspective), or may have fallen short of potential output given its observed level of inputs (output perspective). This theoretical or conceptual ambivalence that the applied researcher faces when choosing a particular orientation has an immediate implication in a cost minimizing analytical framework because a firm, when demanding its optimal input quantities, may take as reference its actually observed output level that the firm has not been capable of producing efficiently by incurring in input excesses (an input perspective), or the intended—and unknown—potential output level (an output perspective).

In this respect, the analytical implications of the choice of the output benchmark are clear. Given the observed market prices for inputs, the first order conditions for cost minimization subject to a given output level determine whether the firm is allocative efficient or not; particularly, if the marginal rates of technical substitution are equal to the price ratios. As a result, a firm will demand different input mixes depending on its *ex-ante* planned output level, which may not be realized latter on resulting in technical inefficiency. Assuming perfect competition in the input markets results in price taking firms, and therefore alternative input mixes imply different allocative efficiency levels. As a result both technical and allocative efficiency will differ depending on the chosen orientation when assessing overall cost efficiency.

Relevant for this discussion, Bogetoft et al. (2006) prove that if the technology is homothetic then both decompositions based on the standard and reversed Farrell approaches are equivalent. Therefore, researchers do not have to worry about whether the subjective analytical choice of orientation yields alternative decompositions of overall economic efficiency, as they are the same. This is because from an economic theory perspective, one remarkable result of homotheticity is that least cost expansion paths are vectors passing through the origin and, therefore, this property preserves marginal rates of substitution as one moves along rays from the origin. Consequently, as it is well-known in the standard Farrell approach, radial measures preserve the value of allocative efficiency along the contracting paths given by the input mix. Since market prices are exogenous, allocative efficiency remains constant along radial projections of technically inefficient firms. As marginal rates of substitution do not change, whatever the difference between the ratios of market prices and marginal rates might be (when they are equal the firm is allocative efficient), it does not change regardless of the input or output isoquant that is considered to evaluate allocative efficiency—formally the marginal rates of substitution are independent of the output levels. Moreover, it is this normally overlooked property of homothetic technologies what guarantees that the radial movements associated to the traditional input and output measures can be rightly interpreted as pure technical efficiency gains, since allocative efficiency remains unchanged, resulting in a consistent decomposition of overall economic efficiency. In this framework, and not surprisingly, Chambers and Mitchell (2001) established the advantages of assuming homotheticity as the most common functional restriction used in economics. Specifically, the level sets for a homothetic function are radial expansions (''blow ups'') of a reference level set.

While non-homotheticity in a parametric setting has been addressed both by the non-frontier and the frontier approaches, it has been completely disregarded by the non-parametric—DEA—approach. From a parametric perspective, classical references showing that non-homothetic parameters for a diversity of functional forms are indeed statistically significant are Christensen and Greene (1976) for a translog cost function, or Sato (1977) for a constant elasticity of substitution specification. In the frontier approach, Koop and Diewert (1982) also allowed for a non-homothetic generalized Cobb-Douglas and translog functional forms. However, even if non-homotheticity is pervasive, its existence goes normally untested among practitioners, who customarily rule it out and therefore do not test if the underlying assumption of homotheticity is supported by the data. From a non-parametric perspective, the issue of whether the most common DEA models are non-homothetic has not been studied until now, including the consequences it has on the decomposition of economic efficiency. Additionally, there are recent attempts of moving stochastic frontier methods to semi-parametric and non-parametric estimation techniques that also allow flexible functional forms—e.g. Delis, Iosifidi, and Tsionas (2014), where the introduction of the ideas developed in this study represents a promising field of research.

One interesting byproduct of the reversed Farrell decomposition proposed by Bogetoft et al. (2006) is that it opens the way to determine allocative efficiency without first projecting the evaluated firm on the isoquant corresponding to the observed level of output. In this respect, a point that has received little attention in the production economics literature and that stems from the above discussion is that if one is interested in measuring the technical efficiency corresponding to a firm producing in the interior of the production possibility set through movements to the frontier, then it is necessary to assure that the allocative efficiency does not change along this process—as in the standard Farrell approach for homothetic technologies. In other words, if we determine the 'starting' allocative efficiency of the assessed firm before projecting it on the frontier of the technology, applying the reversed approach, this value should coincide with the estimation of the allocative efficiency at the projected point after moving the original production plan of the firm to the corresponding isoquant. Only in this way we could be sure that the gains in moving from the original to the projected plan are waste due to exclusively technical reasons. In a homothetic setting researches do not have to worry about how to measure the residual allocative efficiency, either by the standard or reversed approaches since both methodologies coincide, but this would not be the case for non-homothetic technologies. Keeping in mind that true technologies will not generally follow the stylized assumptions underlying theoretical analyses, we believe that to define, interpret and correctly measure technical efficiency, it is necessary to keep constant the allocative efficiency so as to rightly and unambiguously decompose overall efficiency.

As a result of these reflections, in this paper we maintain that the interpretation of the scores in the well-known radial Data Envelopment Analysis models (the CCR by Charnes, Cooper, & Rhodes, 1978 and the BBC by Banker, Charnes, & Cooper, 1984) as technical efficiency is unclear unless we can assume that the underlying technology is homothetic, a scenario that is verified only for the production of one output when the technology exhibits constant returns to scale (CRS). This implies that unless researchers are certain of the mistakes made by the managers of the firm resulting in input excesses or output deficits (and note that individual firms in the evaluated sample could differ in their production errors), the decomposition of overall economic efficiency may be erroneous. Additionally, we propose a simple solution for properly measuring technical efficiency and decomposing overall efficiency when the technology is non-homothetic,