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The value of real time yield information in multi-stage inventory systems – Exact and heuristic approaches

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ABSTRACT

We consider a random yield inventory system, where a company has access to real time information about the actual yield realizations. To contribute to a better understanding of the value of this information, we develop a mathematical model of the inventory system and derive structural properties. We build on these properties to develop an optimal solution approach that can be used to solve small to medium sized problems. To solve large problems, we develop two heuristics. We conduct numerical experiments to test the performances of our approaches and to identify conditions under which real time yield information is particularly beneficial. Our research provides the approaches that are necessary to implement inventory control policies that utilize real time yield information. The results can also be used to estimate the cost savings that can be achieved by using real time yield information. The cost savings can then be compared against the required investments to decide if such an investment is profitable.

1. Introduction

We consider an inventory system, where replenishment orders are subject to random yield. Random yields are an important issue in many procurement, production, and assembly processes (Yano & Lee, 1995). In the food or chemical cold chain, for instance, products are shipped over long distances in refrigerated containers. If the temperature of the product leaves a certain range, the product is spoiled and must be re-ordered. Another example is the semiconductor industry, where production steps are subject to random yield (Wang, 2009).

Recently, technologies have been developed that collect and transmit data about the state of a product in the order pipeline. In cold chains, smart sensors are used to monitor the temperature of products and to inform customers immediately if the temperature leaves a pre-defined range (Zacharewicz, Deschamps, & Francois, 2011). White and Cheong (2012), for instance, consider a food supply chain that requires this type of supply chain visibility. They quantify the benefit of observing the quality of a perishable product that is processed in multiple steps from origin to destination. At each step during the journey the decision has to be made whether or not to inspect the quality of the product at a certain cost and whether or not to continue the transport. More application examples of technologies that enable real time yield information sharing in this context can be found in Hsueh and Chang (2010).

Real time yield information is also relevant in production processes. Consider a supplier that manufactures a product in several production steps, where each step has random yields. The customer of the supplier considers this risk when placing orders with the supplier and therefore determines the input quantity for the supplier's first production step. The supplier holds no inventory (except work in progress) and shares yield information after each production step with the customer. Gavirneni (2004), Inderfurth and Vogelgesang (2013), and Wang (2009) provide details of such a process in the semiconductor industry. Choi, Blocher, and Gavirneni (2008), for instance, consider real time yield information sharing in such a context. However, collecting and transmitting real time yield information requires investments in information technology. To decide whether or not such investments are profitable, the value of using real time yield information must be quantified and we address this topic in this paper.

Research on random yield inventory models can be traced back to Karlin (1958). Karlin (1958) considers a single period inventory system where the yield of an order is a random variable with a known distribution and where order decisions are binary. The structure of the *optimal random yield policy* for inventory systems with zero lead time has been derived by Gerchack, Vickson, and Parlar (1988) and Henig and Gerchak (1990). Gerchack et al. (1988) analyze a finite horizon periodic review problem and show

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that the optimal policy is complex and not myopic. They determine the optimal solution by dynamic programming. Henig and Gerchak (1990) derive structural results for the finite and infinite horizon problems and show that there exists a threshold for each period, such that an order is placed if and only if the on-hand inventory is below the threshold value. They show that the threshold is higher under stochastic yield than under deterministic yield. An overview of periodic review systems with random yield can be found in Yano and Lee (1995).

Because large problems cannot be solved optimally in reasonable time, research has also addressed the development of *random yield heuristics*. Many of these heuristics rely on myopic linear inflation policies (Huh & Nagarajan, 2010). These policies use an order threshold and an inflation factor: If the inventory level is below the order threshold, then the difference between the order threshold and the inventory level multiplied by an inflation factor is ordered. A seminal article in this area is by Bollapragada and Morton (1999). They develop three myopic heuristics that are based on the solution of a newsvendor model with random yield. For a discounted cost model, Li, Xu, and Zheng (2008) develop upper and lower bounds for the optimal order threshold and the order quantity. They use these bounds in a heuristic that outperforms the heuristics of Bollapragada and Morton (1999). Huh and Nagarajan (2010) show how the optimal order threshold of a linear inflation policy can be computed for a given inflation factor.

The existing literature on optimal and heuristic solutions considers models with zero lead time, an assumption under which real time yield information sharing is not an issue. In inventory systems with positive lead times, real time yield information sharing can improve performance. To our best knowledge, Choi et al. (2008) is the only article that analyzes the value of real time yield information sharing in settings with positive lead times. They consider a supply chain with a single supplier and a single manufacturer. The supplier uses a manufacturing process with two processing steps with random yields. Translated to a supply chain setting, their model corresponds to an inventory model with a lead time of three periods, where the first two periods are subject to random yield. To solve the model, Choi et al. (2008) modify one of the heuristics of Bollapragada and Morton (1999).

We also consider a model with positive lead time and allow for an arbitrarily long lead time. Unlike previous research, we derive structural properties of the objective function and prove the existence of a stationary optimal policy for the infinite horizon problem. We show that the objective function is convex and build on this property to optimally solve small and medium sized problems. To solve large problems, we develop two heuristic solution approaches based on linear inflation policies. The first heuristic builds on the MULT-heuristic that was first proposed by Ehrhardt and Taube (1987). The second heuristic is based on the work of Huh and Nagarajan (2010). We provide numerical results that indicate that our heuristics perform well in a variety of settings and we identify conditions under which real time yield information is particularly beneficial.

Related to our research is the research on RFID. For a comprehensive literature review we refer to Lee and Özer (2007), Ngai, Moon, Riggins, and Yi (2008), and Sarac, Absi, and Dauzère-Pérès (2010). For a literature review on applications of RFID technology we refer to Zhu, Mukhopadhyay, and Kurata (2012). To analyze the value of increased supply chain transparency few analytical models have been developed. Our paper derives an analytical model and quantifies the value of real time yield information and we contribute to the filling of the frequently cited credibility gap of the value of RFID (Lee & Özer, 2007; Sari, 2010).

The remainder of the paper is organized as follows. In Section 2, we develop a dynamic program for a periodic review inventory system with random yields. In Section 3, we discretize the state space and use a Markov decision process to compute the optimal solution. In Section 4, we develop heuristic solution approaches. In Section 5, we provide numerical results. In Section 6, we discuss the value of real time yield information in detail. In Section 7, we extend our analysis for the case where fixed order cost is charged. In Section 8, we conclude. All proofs can be found in the appendix.

2. Model formulation

We first consider a supply chain with real time yield information sharing (Section 2.1) and analyze the finite horizon version (Section 2.1.1) and the infinite horizon version (Section 2.1.2) of the problem. We consider both versions of the problem, because each version has properties beneficial in our analyses. For the finite horizon version, we prove the convexity of the value function. We build upon this property to derive the stationary optimal policy for the infinite horizon version, which allows us to compute the optimal expected cost with arbitrary accuracy. One of our objectives is to analyze the value of using real time yield information, which requires us to compare the cost of a supply chain that utilizes real time yield information with the cost of a supply chain that does not utilize this information. Therefore, we also analyze a supply chain without real time yield information (Section 2.2), again for both the finite horizon version and the infinite horizon version of the problem.

2.1. Model with real time yield information

Consider a single manufacturer who places orders with a single supplier. The demand d_t of the product is stochastic and i.i.d. across periods. We denote the order quantity in period t by O_t and orders arrive after a lead time of λ periods. In each lead time period, orders are subject to random yields. The yield rate of lead time period r ($r = 1,\ldots,\lambda$) in period t is $u_{r,t}$. Order $O_{t-\lambda}$ placed in period $t-\lambda$ experiences λ random yields and the replenishment quantity $Q_{t-\lambda}$ in period t is $Q_{t-\lambda} = u_{\lambda,t-1}\, u_{\lambda-1,t-2} \cdots u_{1,t-\lambda} O_{t-\lambda}$. The yield rates $u_{r,t}$ are i.i.d. over time and can be arbitrarily distributed. For ease of presentation, we will drop the index t in $u_{r,t}$ whenever it is appropriate. This yield model is commonly used to analyze the random yield inventory problem, e.g. Choi et al. (2008), Ehrhardt and Taube (1987), and Gerchack et al. (1988).

The sequence of events in each period is as follows: First, the manufacturer observes the current state of the inventory system $z_t = (IL_t, Q_{t-\lambda},\ldots,Q_{t-1})$, which consists of the inventory level IL_t and the current yield of the λ outstanding orders (Fig. 1). Then, the manufacturer decides on the order quantity of the current period and orders, O_t. Next, the manufacturer receives the order that was placed λ periods ago, $Q_{t-\lambda}$. The manufacturer satisfies demand and backorders excess demand. Based on the net inventory IL_{t+1} at the end of period t, backorder or inventory holding costs are charged. With this sequence of events, there are $\lambda+1$ state variables. The intricate dynamics make it impossible to reduce the state to a single variable. All notation is summarized in Appendix A.

2.1.1. Finite horizon model

We formulate the finite horizon version of the optimization problem as a dynamic program. Given the current state z_t, the objective is to determine the order quantities for the current and all future periods, such that the sum of expected inventory holding and backorder costs is minimized:

$$V_t(z_t) = \min_{O_t \geqslant 0} H_t(z_t, O_t), \tag{1}$$

with $H_t(z_t,O_t) = E_{d_t}[C(IL_t + Q_{t-\lambda} - d_t)] + \gamma E_{u_{1,t}} \cdots E_{u_{\lambda,t}} E_{d_t}[V_{t+1}(z_{t+1})].$

γ denotes the discount factor. Without loss of generality, we assume that $V_{T+1}(z_{T+1}) = 0$. The total cost function H_t (z_t, O_t) is the