European Journal of Operational Research 240 (2015) 183–192

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Two consensus models based on the minimum cost and maximum return regarding either all individuals or one individual

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ARTICLE INFO

Article history:
Received 5 January 2014
Accepted 24 June 2014
Available online 3 July 2014

Keywords:
Group decisions and negotiations
Consensus opinion
Minimum cost consensus
Maximum return
Moderator

ABSTRACT

In some important group decision making, a moderator representing the collective interest, who has predetermined, and possesses an effective leadership and strong interpersonal communication and negotiation skills, is crucial to the consensus reaching. In the process of consensus reaching, the moderator needs to persuade each individual to change his/her opinion towards a consensus opinion by paying a minimum cost, while the individuals have to modify and to gradually approach this consensus opinion by expecting to obtain a maximum compensation. This paper, which proposes two kinds of minimum cost models with regard to all the individuals and one particular individual respectively, shows the economic significance of these two models by exploring their dual models grounded in the primal–dual linear programming theory, and builds the conditions under which these two models have the same optimal consensus opinion. The validity of the theoretical analysis is confirmed by numerical examples.

1. Introduction

Group decision making (GDM) (Arrow, 1963; Palomares, Liu, Xu, & Martínez, 2012) requires the subjective judgment of a number of decision makers (DMs) to solve complex and unstructured problems, such as negotiations and conflict resolutions. In the process of GDM, different DMs may represent different interest groups, and may have different values or preferences even they have the same interest. In a GDM, most DMs may eventually arrive at a certain degree of consensus associated with the most relevant alternatives after thought-provocative discussions and many round of negotiations. The consensus decision making (Eklund, Rusinowska, & De Swart, 2007, 2008; French, 1981; Lehrer & Wagner, 1981; Liu & Zhang, 2013; Palomares, Martínez, & Herrera, 2014) is the base of making group choices. In recent years, abundant achievements have been made in the fields of consensus measure and consensus modeling.

1.1. Consensus measure

Consensus measure is mainly about the similarity or dissimilarity among DMs' opinions (preferences, interests). The early literatures suggest a "hard" approach (Bezdek, Spillman, & Spillman, 1978; Spillman, Bezdek, & Spillman, 1979) to measure the consensus level of a group, where the value of consensus level is between 0 and 1. The closer to 1 the index is, the higher consensus level is achieved; and conversely, the closer to 0 the index is, the lower consensus level is. The "hard" approach to consensus modeling is based on the premise that a full agreement within the group has been arrived at, which is also called a Utopian consensus by Tapia García, Del Moral, Martínez, and Herrera-Viedma (2012). It is difficult to achieve such a complete consensus (Cabrerizo, Moreno, Pérez, & Herrera-Viedma, 2010). Kacprzyk and Fedrizzi (1986, 1988, 1989), Kacprzyk and Fedrizzi (1989, 1992, 1997) and Fedrizzi, Kacprzyk, and Nurmi (1993) propose a "soft" method instead of the "hard" approach to measure the consensus level which is also referred as "soft" consensus degree level (Chiclana, Tapia Garcia, del Moral, & Herrera-Viedma, 2013). Since the key elements in GDM are based on human thinking and subjective judgment, most experts only expect to reach a fuzzy-majority-sense consensus at the best. The development of soft decision making theories such as fuzzy decision making theory and linguistic decision making theory provides a rich tool to the "soft" approach-oriented research over recent years (Ben-Arieh & Chen, 2006; Bezdek et al., 1978; Cabrerizo, Alonso, Pérez, & Herrera-Viedma, 2008; Carlsson et al., 1992; Dong, Xu, & Li, 2008; Fedrizzi, Kacprzyk, & Zadrozny, 1988, 1999, 2007; Tapia García et al., 2012; Herrera-Viedma, Martinez, Mata, & Chiclana, 2005; Kacprzyk & Fedrizzi, 1989; Kacprzyk et al., 1997; Parreiras, Ekel, & Morais, 2012; Xu, Wu, & Zhang, 2014; Xu, Li, & Wang,

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http://dx.doi.org/10.1016/j.ejor.2014.06.035
0377-2217/

2013; Xu & Cai, 2013). Usually, "soft" consensus degree level is computed by distance metrics such as Euclidean, Cosine and Jaccard distance functions. Recently, Chiclana et al. (2013) prove that different distance functions have significantly different effects on the speed of achieving consensus by exploring a statistical comparative method.

1.2. Consensus modeling

The optimization consensus modeling is based on the assumption that there exists an optimum consensus opinion such that deviations between this opinion and individual DMs' opinions should be as small as possible. The aggregation model supposes that there exists a suitable aggregation operator that would be able to aggregate all the individual DMs' opinions to the consensus opinion (Ben-Arieh & Easton, 2007; Ben-Arieh, Easton, & Evans, 2009; Dong, Xu, Li, & Feng, 2010; Dong, Li, Xu, & Gu, 2014; Fu & Yang, 2010, 2011, 2012; Xu et al., 2013; Xu, 2009, 2012; Xu & Cai, 2011; Zhang, Dong, Xu, & Li, 2011; Zhang, Dong, & Xu, in press). Technically, consensus models are mostly constructed by using methods of optimization, and belong to the "hard" approach. However, each optimization model is constructed on the assumption that individual DMs' opinions do not exceed a tolerated error of consensus opinion after many times of dynamically revisions and modifications (Bryson, 1996, 1997; Bryson & Joseph, 1999; Dong et al., 2014; Zhang et al., in press). It means that consensus modeling is actually a combination of "soft" and "hard" approaches.

In the last few years, the rapid development of web technologies provides much more convenient platforms for larger number of users from all over the world to freely communicate, share and exchange ideas. Therefore, consensus modeling also needs to incorporate the feedback mechanism during consensus decision making: Alonso, Pérez, Cabrerizo, and Herrera-Viedma (2013) explore a novel linguistic consensus model for Web 2.0 communities, which increases the speed of consensus convergence; Pérez, Cabrerizo, Alonso, and Herrera-Viedma (2014) build up a new consensus model, which specially considers the heterogeneity of DMs; and Pérez, Wikström, Mezei, Carlsson, and Herrera-Viedma (2013) develop a consensus model by using the power of a fuzzy ontology, which deals with the psychology of negotiation. In many consensus decision making, it takes time, requires efforts, and then needs to pay cost to convince DMs to shift their opinions during the feedback process. To model this kind of consensus decision making, Ben-Arieh and Easton (2007) develop a minimum cost consensus model to obtain the optimal convergence point of all DMs: A moderator who represents the collective interest to help reach the consensus is introduced during consensus process, where he/she has been predetermined and possesses an effective leadership and strong interpersonal communication and negotiation skills (Bryson, 1996; Cabrerizo et al., 2008, 2010; Herrera, Herrera-Viedma, & Verdegay, 1996; Herrera-Viedma et al., 2005, Herrera-Viedma, Alonso, Chiclana, & Herrera, 2007, 2014; Mata, Martínez, & Herrera-Viedma, 2009; Palomares et al., 2012, 2014; Pérez et al., 2013; Tapia García et al., 2012). On one hand, the moderator tries his/her best to convince most of the individuals to conform to the collective interest or value by spending all possible forms of resources, such as material, financial, human, and information. He/She always wishes that the amount of resources he/she spends is as small as possible (Ben-Arieh & Easton, 2007; Ben-Arieh et al., 2009; Zhang et al., 2011). On the other hand, every individual DM has an eye on his/her own benefit. Each individual DM hopes that his/her opinion deserves to be particularly considered, or he/she should show the significance and value of himself/herself by playing an important role in the consensus decision making. When they have to change their opinions or they offer more useful opinions, they deserve to be compensated or to be rewarded. Each individual DM always hopes that his/her return is as big as possible. The minimum cost and the maximum return are respectively, the moderator's optimum objective and the individual DMs' optimum objective, and they are dual to each other mathematically, making it helpful to further explore the consensus reaching problem by considering both minimum cost and maximum return.

Considering the moderator's interest, Ben-Arieh and Easton (2007) and Ben-Arieh et al. (2009) suggest a consensus model with linear minimum cost and a consensus models with quadratic cost respectively, to obtain the optimum consensus opinion. Recently, Zhang et al. (2011) and Zhang, Dong, and Xu (2013) generalized Ben-Arieh and Easton's work by proposing a novel consensus model with aggregation operators to obtain the maximum consensus degree under the given cost budget. However, there is few research on consensus model considering the individuals' interests. Actually, the process of consensus reaching needs balancing both the moderator's and the individuals' interests. The theories of primal–dual optimal programming will help to discuss how to obtain an optimum consensus opinion by preserving the benefits of both sides.

This paper discusses two kinds of consensus decision making problems by constructing primal–dual linear programming models. The first is that when all individuals are taken into account as a whole, a primal problem of minimum cost and its dual problem of maximum return for reaching the greatest consensus regarding all the individuals are developed. Secondly, when most individuals' opinions do not exceed the tolerated error (or mathematically, in the neighborhood) of consensus opinion as suggested by the moderator, the individual DMs accept the consensus opinion but expect nothing about the return, while only a few DMs insist on their opinions unless the moderator pays more to them, this means that they accept the consensus opinion conditionally. For convenience, we suppose that there is only one individual who needs to be paid. Hence, a primal problem of minimum cost and its dual problem of maximum return for reaching the greatest consensus regarding one individual are also investigated.

This paper is structured as follows. Section 2 discusses the description of our problem. Section 3 constructs the primal–dual models based on the minimum cost consensus problem and the maximum return regarding all individuals. Section 4 discusses the economic significance of the primal–dual models by introducing its dual properties and exploring their relationship. Similarly, Section 5 establishes the primal–dual models based on the minimum cost and maximum return regarding only one individual, and investigates the economic significance of these models. Section 6 builds the conditions under which these two kinds of primal–dual models have the same optimal consensus opinion. Lastly, conclusion and problems for the future research are provided in Section 7.

2. Problem description

Suppose that there are m decision makers (DMs) $D = \{d_1, \ldots, d_m\}$, that take part in GDM. Let $o_i \in R$ represent the opinion of DM d_i $(i \in M = \{1, 2, \ldots, m\})$ in GDM. Without loss of generality, we always suppose that $o_1 \leqslant o_2 \leqslant \ldots \leqslant o_m$. According to the American Heritage Dictionary, consensus is defined as "an opinion or position reached by a group as a whole". This means in group decision making, the ideal state is that there exists an ideal opinion $\tilde{o}$ such that $o_1 = o_2 = \ldots = o_m = \tilde{o}$. When such an ideal opinion is derived, we get a full and unanimous agreement or a Utopian consensus. However, according to Ness and Hoffman (1998), consensus represents "a decision that has been reached when most members of the team agree on a clear option and the few who oppose it think they have had a reasonable opportunity to influence that choice; all team members agree to support