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Discrete Optimization

A multi-start algorithm for a balanced real-world Open Vehicle Routing Problem

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ABSTRACT

The aim of this paper is to solve a real-world problem proposed by an international company operating in Spain and modeled as a variant of the Open Vehicle Routing Problem in which the makespan, i.e., the maximum time spent on the vehicle by one person, must be minimized. A competitive multi-start algorithm, able to obtain high quality solutions within reasonable computing time is proposed. The effectiveness of the algorithm is analyzed through computational testing on a set of 19 school-bus routing benchmark problems from the literature, and on 9 hard real-world problem instances.

1. Introduction

This work is motivated by a real-world problem that concerns the planning of bus services for an international company operating in Spain where everyday a huge number of employees must be picked up in several locations and taken to their working place.

The company has nine workplaces located in different cities in Spain, hence nine home-to-work bus services have to be optimized. The transport service planning is negotiated between the company management and the unions and approximately 5 million Euros are spent every year by this company for the transportation of employees. Therefore, the main objective of the optimization is the reduction the total cost of the process, which is primarily related to the number of routes (i.e., vehicles) used.

The most important constraint imposed in the real-world problem concerns the time that the employees spend on the bus, which is limited to one hour. This constraint arises since companies in Spain have to protect and insure their employees not only in the workplace but also on the way to it and back to their home. However, insurance companies cover road accidents only if they occur one hour before or after the working hours. Therefore, a maximum duration of one hour is imposed to all routes so that all employees are insured. In addition, *balanced routes*, i.e., having similar duration, are desired to avoid conflicts and uneven treatment for the employees.

Given these characteristics, the problem can be modeled as a variant of the Open Vehicle Routing Problem (OVRP). The OVRP is a well known optimization problem in which either the vehicles are not required to return to the depot after completing their services, or they must return in reverse order along the route. In the OVRP, we are given a set of customers with a specified demand of goods and a depot where a fleet of identical capacitated vehicles is located. We are also given the "traveling costs" between the depot and all the customers, and between each pair of customers. The OVRP then consists in finding a set of routes performed by the vehicles and such that:

1. each route starts at the depot and ends at one of the customers, or vice versa,
2. each customer is served once by exactly one route,
3. total demand served by each route must not exceed the vehicle capacity, and
4. each route must have a total cost not exceeding a prescribed maximum.

Generally, the objective of the classical OVRP is to minimize the number of required routes (i.e., vehicles) first and then the total cost. Some authors address a variant of this problem in which the objective is just minimizing the total cost without taking into account the number of required routes. Many real-world problems fit into the OVRP framework. For example, those companies not owning a vehicle fleet that must contract services to external transporter (see Tarantilis, Kiranoudis, Ioannou, & Prastacos, 2005; Tarantilis, Diakoulaki, & Kiranoudis, 2004), pick up and delivery VRP (see Schrage, 1981) and the planning of train services or bus routes (see Fu, Eglese, & Li, 2005; Bodin, Golden, Assad, & Ball, 1983).

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To take into account the route balancing and the maximum route duration constraint our problem is modeled as an OVRP calling for the minimization of the makespan, i.e., the maximum time spent on the vehicle by a user (see, e.g., Golden, Laporte, & Taillard, 1997). We denote the resulting problem as the *Balanced OVRP* (BOVRP). Clearly, minimizing the makespan leads to a set of routes whose makespan is smaller than the prescribed maximum travel duration, since otherwise the problem is infeasible. Furthermore, as often done in practice, to minimize the number of used vehicles we iteratively solve a BOVRP with fixed number of vehicles and we gradually increase such value until a feasible solution is found.

The OVRP, and thus BOVRP, is an $\mathcal{NP}$-hard combinatorial optimization problem by reduction from the Hamiltonian path problem (see Letchford, Lysgaard, & Eglese, 2007). Hence, it is reasonable to develop an approximate algorithm to find good quality feasible solutions in a reasonable computing time.

In this work a multi-start algorithm specifically designed for the BOVRP is proposed. Multi-start methods have two phases that are executed a certain number of iterations. The first phase generates a feasible solution and the second phase attempts to improve the outcome. They are well-known algorithms that have been successfully applied to a wide variety of complex real-world problems.

The paper is organized as follows: Section 2 describes the problem and provides a comprehensive literature review on methods proposed for BOVRP. Section 3 discusses the proposed algorithm. Then, numerical experiments to test the performance of the algorithm will be presented in Section 4. Finally, in Section 5 conclusions are drawn and further research directions are suggested.

2. Problem description and related work

The OVRP can be formally stated as follows. Let $G = (V, E)$ be a complete graph, where $V = \{0, 1, \ldots, n\}$ is the node set and $E = \{(i,j) : i,j \in V, i \neq j\}$ is the edge set. Node 0 is the depot and $N = \{1, \ldots, n\}$ is the set of customers. Each edge $(i,j) \in E$ has an associated cost c_{ij} and each customer $i \in V$ has a demand $q_i > 0$ (with $q_0 = 0$). Note that in our application the demand corresponds to the number of employees to be picked up at a given location and the cost of an edge is the associated travel time. Let $M = \{1, \ldots, m\}$ be the fleet of m identical vehicles located at the depot. Each vehicle has an associated capacity Q and a maximum cost limit C (i.e., a maximum duration). The customers must be served by m Hamiltonian paths (open routes), each associated with one vehicle, starting at the depot and ending at one of the customers, or vice versa. Each vehicle's route cannot exceed neither the cost limit nor the capacity. The objective of the OVRP is to minimize the total cost of the routes.

As we previously mentioned, in the BOVRP we minimize the makespan instead of the total cost. In addition, the minimization of the number of vehicles is obtained by repeatedly solve the BOVRP with fixed number of vehicles and determine the smallest value for which a feasible solution of BOVRP is found. Figs. 1 and 2 show examples of an OVRP and a BOVRP solution, respectively. According to Fig. 1, these six routes are suitable for an OVRP solution where the total cost is minimized. Specifically, this is a benchmark instance known as C6 in the literature. In the example, the OVRP has a total cost of 412.96 and the longest route is the route number 6 with a cost of 80.72. Solving a BOVRP we get a more balanced set of routes, as depicted in Fig. 2. In the BOVRP solution, the total cost has incremented (432.19) but the longest route is now the route number 3 with a total cost of 77.92 and the cost of the route number 6 is reduced: 77.14.

Most of the existing literature is devoted to OVRP. The first mentioning of OVRP is in a paper by Schrage (1981) in 1981, where an explicit distinction is made between closed trips traveled by private vehicles, and open trips assigned to common carrier vehicles.

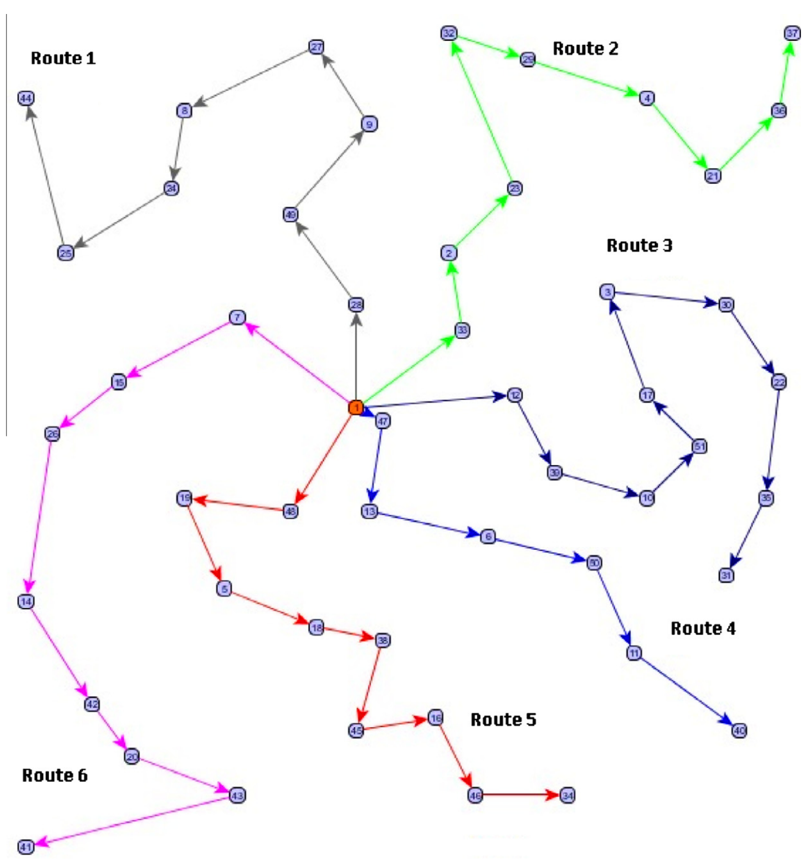

Fig. 1. OVRP feasible solution.