European Journal of Operational Research 238 (2014) 245–253

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

A new nonlinear interval programming method for uncertain problems with dependent interval variables

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ARTICLE INFO

Article history:
Received 28 March 2013
Accepted 18 March 2014
Available online 12 April 2014

Keywords:
Uncertainty modeling
Nonlinear interval programming
Interval model
Uncertain optimization
Variable dependency

ABSTRACT

This paper proposes a new nonlinear interval programming method that can be used to handle uncertain optimization problems when there are dependencies among the interval variables. The uncertain domain is modeled using a multidimensional parallelepiped interval model. The model depicts single-variable uncertainty using a marginal interval and depicts the degree of dependencies among the interval variables using correlation angles and correlation coefficients. Based on the order relation of interval and the possibility degree of interval, the uncertain optimization problem is converted to a deterministic two-layer nesting optimization problem. The affine coordinate is then introduced to convert the uncertain domain of a multidimensional parallelepiped interval model to a standard interval uncertain domain. A highly efficient iterative algorithm is formulated to generate an efficient solution for the multi-layer nesting optimization problem after the conversion. Three computational examples are given to verify the effectiveness of the proposed method.

1. Introduction

In traditional mathematical programming, all of the parameters involved in the problem are given deterministic values. However, in many practical optimization problems, some parameters or coefficients behave with some degree of uncertainty. To date, stochastic programming (Charnes & Cooper, 1959; Kall, 1982; Gao, 2005; Doltsinis & Kang, 2006; Du & Chen, 2004) has been widely used to address the above uncertain optimization problems, in which all the uncertain parameters are quantified through the probability model, and hence, precise probability distributions are required for these parameters. However, in practical applications, sufficient experimental samples regarding the uncertainty are often unavailable or very expensive to collect, and hence, constructing precise probability distributions for the parameters will often be difficult.

In the past two decades, a new type of uncertain optimization method, namely, interval programming, has been developed, which can overcome the above deficiency of stochastic programming to a certain extent. In interval programming, only variation bounds of the parameters are required rather than their precise probability distributions, and thus, many complex engineering problems lacking sufficient experimental samples can be conveniently treated. Some studies have already been conducted and reported in this field. A linear programming problem with interval coefficients in the objective function was investigated (Tanaka, Okuda, & Asai, 1973; Rommelfanger, Hanuscheck, & Wolf, 1989). Using an order relation of the intervals, a linear interval programming problem was converted to a deterministic problem (Chanas & Kuchta, 1996; Chanas & Kuchta, 1996). A linear interval programming method was proposed for problems with intervals in both the objective function and the constraints (Tong, 1994). Based on the concept of an "acceptability index", an interval programming method was suggested to address the inequality constraints using intervals (Sengupta, Pal, & Chakraborty, 2001). Based on the assumption that the interval number follows a uniform distribution between its two bounds, a possibility degree was constructed to address the multi-criteria decision problems (Zhang, Fan, & Pan, 1999). A three-step method (ThSM) was developed to solve the interval linear programming without including infeasible solutions (Huang & Cao, 2011).

In the above-mentioned literature, only linear programming problems with interval coefficients were investigated. Considering that most practical engineering optimization problems are nonlinear and even implicit, in recent years, nonlinear interval programming has attracted increasing attention. The interval uncertainty existing in the nonlinear objective function has been considered, and the uncertain optimization has been converted into a deterministic multi-objective optimization problem (Ma, 2002). The

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http://dx.doi.org/10.1016/j.ejor.2014.03.029
0377-2217/

nonlinear interval programming problem has also been changed to a minimax problem, which was subsequently solved using a hybrid algorithm (Cheng, Dai, & Sun, 2004). An important issue regarding the global optimality of the inner-layer interval optimization was discussed, and some interesting phenomena were revealed (Guo, Bai, Zhang, & Gao, 2009). The Karush–Kuhn–Tucker optimality condition on interval-valued nonlinear programming was investigated (Wu, 2007; Wu, 2009). A nonlinear interval programming model was developed and applied to the planning of a municipal solid waste management system (Wu, Huang, Liu, & Li, 2006). An interval programming model was proposed for a general nonlinear optimization problem with interval parameters in both the objective function and the constraints (Jiang, Han, & Liu, 2007), and some efficient algorithms were further developed for this model (Jiang, Han, & Liu, 2008; Jiang, Han, Guan, & Li, 2007; Jiang, Han, & Li, 2012).

In the current studies on linear and nonlinear interval programming, each uncertain parameter is treated as an isolated interval, and hence, the whole uncertain domain forms a "multi-dimensional box". Thus, the above analysis is actually based on an assumption that all of the interval variables involved are mutually independent. Nevertheless, in practice, it is very important for an analyst to be able to model dependencies because complex dependencies routinely appear among real-world parameters in risk assessments, and these dependencies can have profound impacts on the numerical results of risk calculations (Ferson, Hajagos, Berleant, Zhang, & Tucker, 2004). Complex dependencies are certainly not rare, and they are perhaps as common as nonlinearity in physical systems (Ferson et al., 2004). Therefore, it seems necessary to develop an interval programming model that can address problems with dependent interval variables.

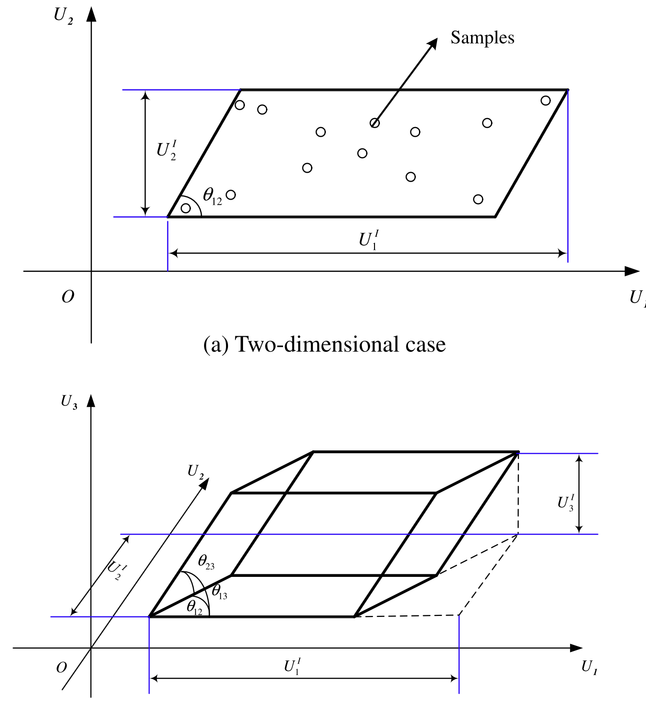
This paper aims to propose a new nonlinear interval programming method based on a multidimensional parallelepiped (MP) interval model, in which the independent and dependent uncertainties can be simultaneously treated very conveniently, and hence, the applicability of interval programming can be significantly improved. The remainder of this paper is organized as follows: Section 2 briefly introduces the MP interval model; Section 3 presents a nonlinear interval programming model to transform the uncertain optimization to a deterministic optimization; Section 4 formulates an efficient algorithm to solve the obtained deterministic optimization problem; Section 5 analyzes three numerical examples; and the conclusions are summarized in Section 6.

2. Multidimensional parallelepiped (MP) interval model

In our recent work (Jiang, Zhang, & Han, submitted for publication), we have proposed a more general nonprobabilistic interval model, i.e., the MP interval model. It can consider the independent and dependent variables simultaneously and represent the uncertainty for all the interval variables under a unified framework. We only need to know the variation intervals for each parameter and the dependencies between any two parameters to construct the uncertain domain Ω which geometrically is a multi-dimensional parallelepiped. Using the two-dimensional problem illustrated in Fig. 1(a) as an example, the MP interval model would degenerate into a parallelogram. For this model, one side of the quadrilateral is set parallel to the axis of abscissas. U_1^I and U_2^I are the individual variation ranges for two uncertain parameters, which are called marginal intervals in this paper:

$$U_i^I = \left[U_i^L, U_i^R\right],\ U_i^c = \frac{U_i^L + U_i^R}{2},\ U_i^w = \frac{U_i^R - U_i^L}{2},\ i = 1,2 \tag{1}$$

where the superscripts and subscripts L, R, c, and w represent the lower bound, upper bound, midpoint and radius, respectively. The



(a) Two-dimensional case

(b) Three-dimensional case

Fig. 1. Multidimensional parallelepiped interval model.

θ_{12} is called correlation angle, which depicts the dependence between the variables U_1 and U_2, and it has the following range:

$$\theta_{12} \in \left[\arctan\frac{U_2^w}{U_1^w}, \pi - \arctan\frac{U_2^w}{U_1^w}\right] \tag{2}$$

When $\theta_{12} < 90°$, the variables U_1 and U_2 are positively correlated; when $\theta_{12} = 90°$, the variables $U_1$and U_2 are independent from each other, and the parallelogram model degenerates into a traditional interval model; when $\theta_{12} > 90°$, the variables U_1 and U_2 are negatively correlated.

For a general q-dimensional problem, the uncertain domain is a multi-dimensional parallelepiped. In Fig. 1(b), a three-dimensional MP interval model is illustrated, in which one surface of the parallelepiped is set parallel to the X–Y coordinate plane. The marginal intervals for different variables are $U_i^I,\ i = 1, 2, \ldots, q$, and the relevant angle between any two variables U_i and U_j is θ_{ij}. To easily depict the dependencies among the interval variables, the correlation coefficient ρ_{ij} between U_i and U_j is defined as:

$$\rho_{ij} = \frac{U_j^w}{U_i^w \tan\theta_{ij}} \tag{3}$$

where $\theta_{ij} \in \left[\arctan\frac{U_j^w}{U_i^w}, \pi - \arctan\frac{U_j^w}{U_i^w}\right]$, and obviously, the range of the correlation coefficients is $[-1, 1]$. As shown in Fig. 2, when $\theta_{ij} = \arctan\frac{U_j^w}{U_i^w}$, the correlation coefficient $\rho_{ij} = 1$, indicating that U_i and U_j are completely linearly positively correlated; when $\theta_{ij} = 90°$, the correlation coefficient $\rho_{ij} = 0$, and the two interval variables are independent; when $\theta_{ij} = \pi - \arctan\frac{U_j^w}{U_i^w}$, the correlation coefficient $\rho_{ij} = -1$, and U_i and U_j are completely linearly negatively correlated. From the above analysis, we know that when the correlation coefficients between all pairs of variables are all zero, the MP interval model degenerates to a traditional interval model, i.e., the interval model is a special case of the MP interval. For the above MP interval model, the marginal intervals and correlation coefficients can all be easily obtained using experiment samples (Jiang et al., submitted for publication).