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# Solving real-world cutting stock-problems in the paper industry: Mathematical approaches, experience and challenges

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ABSTRACT

We discuss cutting stock problems (CSPs) from the perspective of the paper industry and the financial impact they make. Exact solution approaches and heuristics have been used for decades to support cutting stock decisions in that industry. We have developed polylithic solution techniques integrated in our ERP system to solve a variety of cutting stock problems occurring in real world problems. Among them is the simultaneous minimization of the number of rolls and the number of patterns while not allowing any overproduction. For two cases, CSPs minimizing underproduction and CSPs with master rolls of different widths and availability, we have developed new column generation approaches. The methods are numerically tested using real world data instances. An assembly of current solved and unsolved standard and non-standard CSPs at the forefront of research are put in perspective.



## 1. Introduction

The pulp and paper industry plays an important role worldwide. There are in the order of 3000 paper mills, which produced a total of 394 million tons of paper and paperboard, in 2010. Europe (including Russia) has approximately 900 paper mills, while Germany has about 180. The largest producer in the world is the Finnish UPM group with an annual tonnage of 12.7 million tonnes, followed by Stora Enso with 11.8 million tons and by International Paper with 9.7 million tonnes per year. Santos and Almada-Lobo (2012) report that in Portugal the pulp and paper industry contributes over 4% of the GDP and 5% of the active employees. As it is subject of both local and global environmental discussions, effective planning and cutting stock techniques lies at the very heart of the operational performance of its manufacturing organizations.

Exact solution approaches and heuristics have been used for decades to support cutting stock decisions in the paper industry. In the standard cutting stock problem (CSP), the problem input is given by a set of item sizes and demands, and by a set of master rolls of given widths; the simplest case consists of only one type of master rolls. The task is to decide on how many master rolls are cut to a certain pattern in order to minimize the total number of master rolls used.

The pattern minimization problem (PMP) is a strongly NP-hard cutting problem, which seeks a cutting plan with the minimum number of different patterns, *cf.* McDiarmid (1999). This objective, relevant when changing from one pattern to another, involves a cost for setting up the cutting machine, *i.e.*, adjusting the cutting knifes. When the minimization of the number of different patterns is done by assuming that no more than the minimum number of rolls can be used, the problem is also referred to as the cutting stock problem with setup costs.

The international working group SICUP (*Special Interest Group on Cutting and Packing*) founded by Gerhard Wäscher in 1988, focuses on cutting stock and packing problems and is a platform for more than 200 practitioners and scientists to exchange ideas on these topics. In 2004, SICUP became the EURO working group ESICUP (*EURO Special Interest Group on Cutting and Packing*).

The main contributions of this paper can be classified into two categories:

**Mathematical optimization:**
For 1D CSPs with two criteria, minimizing the number of rolls and the number of patterns, we develop an Exhaustion Method

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(Section 3.4), a column generation approach allowing underproduction (Section 3.5.4) and column generation approach incorporating master rolls with different widths and limited availability (Section 3.5.5). We present a novel polylithic[1] solution method towards 2D trim-loss minimization (Section 4). Furthermore, we share real data in a 1D cutting stock benchmark data set (Section 3.6.1). For software products, it is not untypical to combine various basic algorithms to consistently provide solutions in acceptable time, with many empirical rules, or even rules of thumb, to decide which algorithms to use in each circumstance. We disclose this information instead of keeping it as a commercial secret, to provide evidence that there is more exact optimization and less heuristics involved as one might expect.

**Managerial insights for the paper industry:**
We present real-world aspects relevant to the paper industry, which have seen only little treatment in the scientific literature (Section 3.5). We assemble current cutting-edge standard and non-standard cutting stock problems relevant to the paper industry (Section 5) and illuminate at length the variants and issues present in real-world problems. We discuss the financial impact mathematical programming-based solutions to cutting stock problems have in the paper industry (Section 6).

The remainder of the paper is structured as follows: After a literature review in Section 2, we discuss the 1D CSP and its variants in Section 3 along with different solution techniques. A presentation of 2D polylithic solution methods in Section 4 is followed by a discussion of current-edge CSPs in Section 5 and our views on optimization in the paper industry in Section 6. Conclusions are in Section 7. Two appendices, post-processing (Appendix A) and and guidelines on how to derive the pricing problems (Appendix B) complete this paper.

## 2. Literature review

There is a rich body of literature available on CSPs; *cf.* Haessler and Sweeney (1991) and Haessler (1992) for reviews on 1D cutting stock problems and solution procedures. We find heuristic solution approaches (*cf.* Haessler (1971)), exact MILP-models (*cf.* Johnston & Sadinlija (2004)), column generation approaches, among them the classical paper by Gilmore and Gomory (1961), Branch&Price algorithms (*cf.* Belov & Scheithauer (2006)), reviews as by Amor (2005) who put column generation and Branch&Price algorithms in perspective, and classification papers (*cf.* Dyckhoff (1990) and Wäscher, Haußner, & Schumann (2007)).

Most of the approaches described in the literature for solving the PMP are based on heuristics. As the PMP has been proven strongly NP-hard by McDiarmid (1999), it is not a surprise that solving the problem exactly has been a real challenge, and only very few exact solution methods have been reported so far in the literature; among them Vanderbeck (2000). Alves, Macedo, and de Carvalho (2009) explore an integer programming model that can be solved using column generation, and they describe different strategies to strengthen it, among which are constraint programming and new families of valid inequalities. Lower bounds for the pattern minimization problem are derived from the new integer programming model, and also from a constraint programming model.

Beyond a vast body of literature on the standard CSP, there are a few publications on a generalized CSP with great practical significance: The multiple-width CSP with master rolls of different widths (and equal lengths assumed to be infinite). An early work on this topic is by Holthaus (2002), who solves the relaxation of the CSP by the column generation technique and uses three procedures for rounding the solution, leading in a final residual problem, which is solved by an ILP-solver. Although his technique is suitable for solving medium-size and large instances of the one-dimensional CSP, the paper does not consider supply limitation on the different stock lengths availability. Alves and de Carvalho (2007) developed strategies to stabilize and accelerate the column generation method by introducing dual-optimal inequalities, reducing the number of column generation iterations and run time. Finally, Poldi and Arenales (2009) provide a heuristic to solve the CSP with multiple stock lengths with limited availability.

Although production planning or scheduling and CSPs are usually treated separately, we find early articles in which both aspects are combined; *cf.* Haessler and Talbot (1983) or Li (1996) who provide LP-based and non-LP-based heuristics to solve 2D multi-job cutting stock problems with due dates and release dates. The combined cutting stock and lot-sizing problem in industrial processes has attracted several authors in the last decade, among them Arbib and Marinelli (2005), Gramani and França (2006), Yanasse and Pinto Lamosa (2007), Trkman and Gradisar (2007), Poltroniere, Poldi, Toledo, and Arenales (2008), Gramani, França, and Arenales (2009) and most recently Reinertsen and Vossen, 2010 who treat the 1D CSP with due dates. Trkman, Erjavec, and Gradisar (2009) treat cutting stock as a continuous business process which is incorporated into an entire supply chain.

General cutting and packing problems are related to CSPs. The most important difference between cutting and packing problems is that in cutting problems, the number of objects are given and the task is to minimize trim-loss or area, while packing problems aim to fit as many objects as possible in a predefined area or volume. For example, one may want to cut orientation free polygons (Kallrath, 2009b) or ellipses (Kallrath & Rebennack, 2013) into one rectangle, or circles into several rectangles (Rebennack, Kallrath, & Pardalos, 2009). A significant difference between these cutting problems cited and the 2D cutting problems described in Section 4 is that the latter allow only a horizontal or vertical orientation of the objects to be cut.

We conclude our literature review by pointing the reader to a few articles which give some excellent insights into the field: Rodríguez and Vecchietti (2008) for practical application with very good illustrations, and similarities to our 2D problem described in Section 4, Harjunkoski, Westerlund, Pörn, and Skrifvars (1998) and Pörn, Harjunkoski, and Westerlund (1999) for exact MILP and MINLP approaches, and also, a very recent paper on heuristics by Cui and Zhao (2013).

## 3. 1D cutting stock problem

Our discussion of the one-dimensional cutting stock problem starts with the standard problem formulation in Section 3.1, followed by three solution methods: the widely used approach by Gilmore and Gomory (Section 3.2), a column enumeration (Section 3.3), and an Exhaustion Method (Section 3.4). We summarize important practical aspects for one-dimensional CSPs for the paper industry and present extensions to the column generation approach addressing these practical aspects (Section 3.5). We conclude this section with some computational benchmarking (Section 3.6).

### 3.1. The standard problem and its mathematics

The mathematical model for minimizing the number of rolls or trim-loss in the standard problem with one master roll of width $B$ is characterized by the following indices, data and variables.

[1] The term *polylithic* has been coined by Kallrath (2009a) and explained in greater detail in Kallrath (2011); it refers to modeling and solution approaches in which mixed integer or nonconvex nonlinear optimization problems are solved by tailor-made methods involving several models and/or solve statements or algorithmic components.