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Invited Review

The one-dimensional cutting stock problem with usable leftovers – A survey

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ABSTRACT

In this article, we review published studies that consider the solution of the one-dimensional cutting stock problem (1DCSP) with the possibility of using leftovers to meet future demands, if long enough. The one-dimensional cutting stock problem with usable leftovers (1DCSPUL) is a problem frequently encountered in practical settings but often, it is not dealt with in an explicit manner. For each work reviewed, we present the application, the mathematical model if one is proposed and comments on the computational results obtained. The approaches are organized into three classes: heuristics, item-oriented, or cutting pattern-oriented.

1. Introduction

The cutting stock problem consists of cutting a set of parts available in stock (called objects) to produce smaller pieces (called items) in specified quantities, optimizing an objective function. Examples of objective functions include minimizing the total waste, minimizing the cost of cutting the objects, minimizing the total number of objects cut, maximizing profit and minimizing production costs.

One solution to the cutting stock problem, often called the cutting plan, is provided by a set of cutting patterns and their corresponding frequencies, in other words, how many times each cutting pattern must be cut to produce the items. A cutting pattern defines a subset of items to be cut from an object. In the case where two or more dimensions of the items are relevant for the cutting problem, a cutting pattern also includes the layout of the items to be cut in the object.

The cutting stock problem arises in many industrial processes where objects can be steel bars, rolls of paper or aluminum, wood boards or metal sheets, printed circuit boards, glass or fiber glass sheets, leather pieces and others. In these industries, reducing production costs and/or improving efficiency are often associated with using appropriate cutting plans and/or cutting strategies.

The economic importance of such problems and the difficulties in solving them have motivated the operations research community to develop more efficient methods to solve them. Various articles about the cutting stock problem can be found in the literature, as can be seen in review articles and special editions: Hinxman (1980), Dyckhoff, Kruse, Abel, and Gal (1985), Dyckhoff (1990), Dyckhoff and Wäscher (1990), Dyckhoff and Finke (1992), Dowsland and Dowsland (1992), Sweeney and Parternoster (1992), Martello (1994a, 1994b), Bischoff and Wäscher (1995), Dyckhoff, Scheithauer, and Terno (1997), Arenales, Morabito, and Yanasse (1999), Wang and Wäscher (2002), Hifi (2002), Oliveira and Wäscher (2007), Wäscher, Haußner, and Schumann (2007) and Morabito, Arenales, and Yanasse (2009). Additional references may be found in ESICUP (2013).

Each practical situation where cutting stock problems arise has its specific features, constraints and objectives, which often prevents the application of existing models and algorithms in a straightforward manner.

One problem often encountered is the use of leftovers of cutting patterns. This problem has been addressed directly in the literature only recently, although it was cited in the early 1970s by Brown (1971). The cutting stock problem with usable leftovers was studied by Arbibi, Marinelli, Rossi, and Di Iorio (2002) in an automobile industry. In this industry, the items cut were used to produce belts with a fixed width and different lengths. Retails (i.e., the usable leftovers) remaining during the cutting process could be stitched and used to manufacture other different goods. According to the authors, with the possibility of using leftovers, a considerable

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amount of material could be saved. This type of use of leftovers differs from that considered in the papers of this survey. In this paper, the leftovers generated during the cutting process are used directly without further processing.

In this paper, a literature review was carried out on the 1DCSP with leftovers that may be used to produce items demanded in the future, if the leftovers are long enough. We present the application, the mathematical model if one is proposed, as well as comments on the computational results obtained. It should be mentioned that literature on the two-dimensional case is scarce. The interested reader may refer to Cherri (2009), Cherri and Vianna (2010) and Andrade, Birgin, and Morabito (2013).

In addition to published studies, the use of leftovers is also considered in some commercial software available on the Internet, however many of them have limited free versions. Minimizing waste is usually the main objective considered in the software, and several additional restrictions can be inserted. A list of software that considers this problem can be found in Macedo et al. (2008).

The articles considered in this survey address cutting stock problems with usable leftovers. These problems cannot be classified and organized according to the typology proposed by Wäscher et al. (2007). Some of the problems are input optimization, others are output minimization, some consider multiple objects in stock, others just a single type of object, among other characteristics. Since there is no classification that applies to all the problems with these diverse characteristics, this class of problems will be referred to as 1DCSPUL: one dimensional cutting stock problem with usable leftovers.

For the sake of presentation, the revised papers were classified into three groups according to the structure of the mathematical model used in the study or the solution method. The first group contains articles that solve the problem using heuristics not based on mathematical programming models; the second group includes studies that solve the problem using item allocation oriented modeling; the third group contains articles that solve the problem using cutting pattern oriented approaches. In the case whereby an article presents a mixture of model structures or a model combined with heuristics, the feature considered more relevant in the work was used to classify it.

This paper is organized as follows. In Section 2, notation used to present the mathematical models is introduced; in Section 3, studies that use heuristic methods not based on mathematical programming models to solve the 1DCSPUL are presented; in Section 4, studies with item allocation oriented modeling are presented; in Section 5, studies that use models with cutting pattern oriented approaches are presented; and in Section 6, we comment on the different approaches and we present some concluding remarks.

2. General mathematical notation and terminology

In this paper, we will use the term "*standard objects*" to refer to objects purchased on the market and the term "*retails*" to refer to leftovers of previous cuts (not counted as waste) that are long enough to cut items for future usage.

The notation used in the models presented is defined as follows:

Indices

- i: item type;
- j: cutting pattern;
- k: object type.

Stock

- K: number of types of objects in stock;
- L_k: length of object type k;
- e_k: availability of object type k in stock;
- c_k: unit cost of object type k.

Items

- m: number of item types;
- ℓ_i: length of item type i;
- d_i: demand for item type i.

Other parameters

- N_k: total number of cutting patterns related to object type k;
- δ_k: threshold length for a retail from object type k;
- α_{ijk}: number of items type i in cutting pattern j of object type k.

Variables

- x_{jk}: number of objects type k cut according to cutting pattern j (frequency);
- p_{ik}: number of items type i cut from object type k;
- $s_k = L_k - \sum_{i=1}^{m} \ell_i p_{ik}$: leftover from object type k.

This notation considers several types of objects available in stock. When only one type of object is considered in the formulation, index k is omitted. In addition to this general notation, other variables and parameters are defined as needed.

3. Heuristic methods not based on mathematical programming models

Gradisar, Jesenko, and Resinovic (1997) studied the 1DCSPUL in a clothing industry, where the lengths of the objects in stock were all different. To solve this problem, the authors proposed a bi-objective model: minimization of the number of items whose demand are not satisfied and the total trim loss. The possibility of unused pieces being returned to stock was considered. They proposed a mathematical model to the cutting problem with usable leftovers, but the authors did not use it to solve the problem. Instead, they developed a greedy heuristic procedure, called COLA (Computerized Laying out). In COLA, objects are sorted in non-decreasing order and for each object in the sequence, a cutting pattern is built considering three different ways to sort the items. Cutting patterns are generated solving a few knapsack problems (the utility value is changed for each ordering of the items) and the cutting pattern with the lowest trim loss is used. Although computational tests were not presented in the article, the authors claim that they were carried out and COLA algorithm showed a good performance.

In Gradisar, Kljajic, Resinovic, and Jesenko (1999) the authors improved and generalized the heuristic proposed in Gradisar et al. (1997) to solve the 1DCSPUL. Two possible cases were analyzed: the stock is sufficient to meet all demands and the demands cannot be met with the available stock. For each case, a mathematical model is proposed, however once again they are not used to solve the problems. They improved algorithm COLA and denoted it as CUT. According to the authors, CUT finds better solutions (with less trim loss) in a lower computational time.

In Gradisar and Trkman (2005) the authors limited the quantity of retails generated during the cutting process and proposed a combination of the heuristic procedure CUT with a *branch and bound* method to solve the problem.

Dimitriadis and Kehris (2009) presented a study carried out in a Greek customized door and window manufacturing industry. The problem was the classic cutting stock problem with the objective