European Journal of Operational Research 236 (2014) 445–452

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Minimizing earliness and tardiness costs in stochastic scheduling

Kenneth R. Baker *

Tuck School of Business, Dartmouth College, Hanover, NH 03755, United States

ARTICLE INFO

Article history:
Received 5 July 2013
Accepted 5 December 2013
Available online 12 December 2013

Keywords:
Sequencing
Stochastic scheduling
Earliness/tardiness
Branch and bound
Neighborhood search
Due-date assignment

ABSTRACT

We address the single-machine stochastic scheduling problem with an objective of minimizing total expected earliness and tardiness costs, assuming that processing times follow normal distributions and due dates are decisions. We develop a branch and bound algorithm to find optimal solutions to this problem and report the results of computational experiments. We also test some heuristic procedures and find that surprisingly good performance can be achieved by a list schedule followed by an adjacent pairwise interchange procedure.

1. Introduction

The single-machine sequencing model is the basic paradigm of scheduling theory. In its deterministic version, the model has received a great deal of attention from researchers, leading to problem formulations, solution methods, scheduling insights, and building blocks for more complicated models. Extending that model into the realm of stochastic scheduling is an attempt to make the theory more useful and practical. However, progress in analyzing stochastic models has been much slower to develop, and even today some of the basic problems remain virtually unsolved. One such case is the stochastic version of the earliness/tardiness (E/T) problem for a single machine.

This paper presents a branch and bound (B&B) algorithm for solving the stochastic E/T problem with normally-distributed processing times and due dates as decisions. This is the first appearance of a solution algorithm more efficient than complete enumeration for this problem, so we provide some experimental evidence on the algorithm's computational capability. Although B&B algorithms are not new, they have seldom been applied to problems in stochastic scheduling. Arguably, too little research has been done on the application of such optimization approaches in stochastic scheduling problems, so a broader goal of this paper is to demonstrate that methodologies common in deterministic scheduling can successfully be applied to problems in stochastic scheduling.

In addition, we explore heuristic methods for solving the problem, and we show that a relatively simple procedure can be remarkably successful at producing optimal or near-optimal solutions. These results reinforce and clarify observations made in earlier research and ultimately provide us with a practical method of solving the stochastic E/T problem with virtually any number of jobs.

In Section 2 we formulate the problem under consideration, and in Section 3 we review the relevant literature. In Section 4, we describe the elements of the optimization approach, and we report computational experience in Section 5. Section 6 deals with heuristic procedures and the corresponding computational tests, and the final section provides a summary and conclusions.

2. The problem

In this paper we study the stochastic version of the single-machine E/T problem with due dates as decisions. To start, we work with the basic single-machine sequencing model (Baker & Trietsch, 2009a). In the deterministic version of this model, n jobs are available for processing at time 0, and their parameters are known in advance. The key parameters in the model include the processing time for job j (p_j) and the due date (d_j). In the actual schedule, job j completes at time C_j, giving rise to either earliness or tardiness. The job's earliness is defined by $E_j = \max\{0, d_j - C_j\}$ and its tardiness by $T_j = \max\{0, C_j - d_j\}$. Because the economic implications of earliness and tardiness are not necessarily symmetric, the unit costs of earliness (denoted by α_j) and tardiness (denoted by β_j) may be different. We express the objective function, or total cost, as follows:

$$G(d_1, d_2, \ldots, d_n) = \sum_{j=1}^{n} (\alpha_j E_j + \beta_j T_j) \qquad (1)$$

---

* Tel.: +1 6036462064.

E-mail address: ken.baker@dartmouth.edu

0377-2217/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2013.12.011

The deterministic version of this problem has been studied for over 30 years, and several variations have been examined in the research literature. Some of these variations have been solved efficiently, but most are NP-Hard problems. In the stochastic E/T problem, we assume that the processing times are random variables, so the objective becomes the minimization of the expected value of the function in (1). The stochastic version of the E/T problem has not been solved.

To proceed with the analysis, we assume that the processing time p_j follows a normal distribution with mean μ_j and standard deviation σ_j and that the p_j values are independent random variables. We use the normal because it is familiar and plausible for many scheduling applications. Few results in stochastic scheduling apply for arbitrary choices of processing time distributions, so researchers have gravitated toward familiar cases that resonate with the distributions deemed to be most practical. Several papers have addressed stochastic scheduling problems and have used the normal distribution as an appropriate model for processing times. Examples include Anderson and Moodie (1969), Balut (1973), Cai and Zhou (2007), Jang (2002), Portougal and Trietsch (2006), Sarin, Erdel, and Steiner (1991), Seo, Klein, and Jang (2005), Soroush (1999), Soroush and Fredendall (1994), and Wu, Brown, and Beck (2009).

In our model, the due dates d_j are decisions and are not subject to randomness. The objective function for the stochastic problem may be written as

$$H(d_1, d_2, \ldots, d_n) = E[G(d_1, d_2, \ldots, d_n)] = \sum_{j=1}^{n} (\alpha_j E[E_j] + \beta_j E[T_j]) \qquad (2)$$

The problem consists of finding a set of due dates and a sequence of the jobs that produce the minimum value of the function in (2).

3. Literature review

The model considered in this paper brings together several strands of scheduling research – namely, earliness/tardiness criteria, due-date assignments, and stochastic processing times. We trace the highlights of these themes in the subsections that follow.

3.1. Earliness/tardiness criteria

Scheduling problems comprising both earliness costs and tardiness costs were first examined for the case in which processing times and due dates are given. This type of problem was first studied by Sidney (1977), who analyzed the minimization of maximum cost and by Kanet (1981), who analyzed the minimization of total absolute deviation from a common due date, under the assumption that the due date is late enough that it does not impose constraints on sequencing choices. This objective is equivalent to an E/T problem in which the unit costs of earliness and tardiness are symmetric and the same for all jobs. For this special case, Hall, Kubiak, and Sethi (1991) developed an optimization algorithm capable of solving problems with hundreds of jobs, even if the due date is restrictive. In addition, Hall and Posner (1991) solved the version of the problem with symmetric earliness and tardiness costs that vary among jobs. Their algorithm handles over a thousand jobs.

The case of distinct due dates is somewhat more challenging than the common due-date model. Garey, Tarjan, and Wilfong (1988) showed that the E/T problem with distinct due dates is NP-Hard, although for a given sequence, the scheduling of idle time can be determined by an efficient algorithm. Optimization approaches to the problem with distinct due dates were proposed and tested by Abdul-Razaq and Potts (1988), Ow and Morton (1989), Yano and Kim (1991), Azizoglu, Kondakci, and Kirca (1991), Kim and Yano (1994), Fry, Armstrong, Darby-Dowman, and Philipoom (1996), Li (1997), and Liaw (1999). Fry et al. addressed the special case in which earliness costs and tardiness costs are symmetric and common to all jobs. Their B&B algorithm was able to solve problems with as many as 25 jobs. Azizoglu et al. addressed the version in which earliness costs and tardiness costs are common, but not necessarily symmetric, and with inserted idle time prohibited. Their B&B algorithm solved problems with up to 20 jobs. Abdul-Razaq and Potts developed a B&B algorithm for the more general cost structure with distinct costs but prohibited inserted idle time. Their algorithm was able to solve problems up to about 25 jobs. Li proposed an alternative lower bound calculation for the same problem but still encountered computational difficulties in solving problems larger than about 25 jobs. Liaw's subsequent improvements extended this range to at least 30 jobs.

Because optimization methods have encountered lengthy computations times for problems larger than about 25–30 jobs, much of the computational emphasis has been on heuristic procedures. Ow and Morton were primarily interested in heuristic procedures for a version of the problem that prohibits inserted idle time, but they utilized a B&B method to obtain solutions (or at least good lower bounds) to serve as a basis for evaluating their heuristics. They reported difficulty in finding optimal solutions to problems containing 15 jobs. Yano and Kim compared several heuristics for the special case in which earliness and tardiness costs are proportional to processing times. The B&B algorithm they used as a benchmark solved most of their test problems up to about 16 jobs. Kim and Yano developed a B&B algorithm to solve the special case in which earliness costs and tardiness costs are symmetric and identical. Their B&B algorithm solved all of their test problems up to about 18 jobs. Lee and Choi (1995) reported improved heuristic performance from a genetic algorithm. To compare heuristic methods, they used lower bounds obtained from CPLEX runs that were often terminated after 2 hours of run time, sometimes even for problems containing 15 jobs. James and Buchanan (1997) studied variations on a tabu-search heuristic and used an integer program to produce optimal solutions for problems up to 15 jobs.

Detailed reviews of this literature have been provided by Kanet and Sridharan (2000), Hassin and Shani (2005) and M'Hallah (2007). The reason for emphasizing problem sizes in these studies, although they may be somewhat dated, is to contrast the limits on problem size encountered in studies of the distinct due-date problem with those encountered in the common due-date problem. This pattern suggests that stochastic versions of the problem may also be quite challenging when each job has its own due date.

3.2. Due-date assignments

The due-date assignment problem is familiar in the job shop context, in which due dates are sometimes assigned internally as progress targets for scheduling. However, for our purposes, we focus on single-machine cases. Perhaps the most extensively studied model involving due-date assignment is the E/T problem with a common due date. The justification for this model is that it applies to several jobs of a single customer, or alternatively, to several subassemblies of the same final assembly. The E/T problem still involves choosing a due date and sequencing the jobs, but the fact that only one due date exists makes the problem intrinsically different from the more general case involving a distinct due date assignment for each job. Moreover, flexibility in due-date assignment means that the choice of a due date can be made without imposing unnecessary constraints on the problem, so formulations of the due date assignment problem usually correspond to the common due-date problem with a given but nonrestrictive due date.

Panwalkar, Smith, and Seidmann (1982) introduced the due-date assignment decision in conjunction with the common