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A dynamic paired comparisons model: Who is the greatest tennis player?

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ABSTRACT

We present a methodology for fitting time-varying paired comparisons models in which the parameters are allowed to vary deterministically, as opposed to stochastically, with time. Our dynamic paired comparisons model is based on a new closed-form for Stern's continuum of paired comparisons models which include the Bradley–Terry model and the Thurstone–Mosteller model. The dynamic element of our model is facilitated by utilising barycentric rational interpolants BRIs. An incidental result of our work is to show that BRIs often provide a better fit to data than the obvious alternative of spline interpolation. We use our model to shed light on the debate of who is the greatest tennis player of the Open Era of men's professional tennis since 1968. Constructing a single rankings list from our model is not trivial as there are many alternative metrics that could be used to identify which player was the best ever. We present three alternative rankings lists derived from our model. In general our rankings lists largely agree with the rankings list based on number of Grand Slam titles won, which, to some extent, validates our choice of metrics. So who is the greatest tennis player of the Open Era? Roger Federer seems like the most likely candidate, with Bjorn Borg and Jimmy Connors close behind.

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1. Introduction

Rankings models arise in many applications in quantitative analysis. From assessing University league tables, to modelling individual choice behaviour (Luce, 1959) to rankings of rival candidates in elections (Gormley & Murphy, 2008) to ranking countries' performances at the Olympics (Sitarz, 2012), applications of rankings models are widespread. In this paper we concentrate on models for paired comparisons (when many competitors compete in a series of head-to-head competitions) and extend these models to allow for time-varying competitor strengths.

Paired comparisons models are formulated so that each competitor being compared is associated with a strength parameter, α. The probability of one competitor beating another is then given by a function of the ratio of the strength parameters of the two competitors in question. More generally, the probability, p_{ij}, that competitor i beats competitor j, is given by $p_{ij} = F(\mu_i - \mu_j)$, where $\mu_i = \ln(\alpha_i)$, and F is a distribution function. The Bradley–Terry (BT) model (Bradley & Terry, 1952) assumes a logistic distribution for F, and the Thurstone–Mosteller (TM) model (Thurstone, 1927) uses a normal distribution. In terms of the strength parameters, the formulae are respectively $p_{ij} = \frac{\alpha_i}{(\alpha_i+\alpha_j)}$, and $p_{ij} = \Phi\left(\ln\left(\frac{\alpha_i}{\alpha_j}\right)\right)$, where Φ is the standard normal distribution function.

Stern (1990) gives a formula for the probability of a ranking based on a gamma distribution for the underlying times to an event. These times can be thought of as the time for each competitor to run a race in which the quickest wins. One can also consider the random variable to be a score, where the player with the lowest score wins, as in golf. For Stern's model,

$$p_{ij} = \int_0^\infty f_i(x) S_j(x)\,dx = \int_0^\infty F_i(x) f_j(x)\,dx, \quad (1)$$

where f is the pdf $f_i(x) = \alpha_i(\alpha_i x)^{\beta-1}\exp(-\alpha_i x)/\Gamma(\beta)$ of the gamma distribution, S_j the survival function and F_i the distribution function. When $\beta = 1$ the model reduces to the BT model. Stern (1990) shows how his 'gamma comparison model' interpolates between the BT model and the TM model as $\beta \to \infty$ and refers to the model as a *continuum of paired comparison models*. This is clearly a useful model, but Stern (1992) concludes that for samples of the size usually encountered, predictions are not sensitive to the value of β. However, as we will see, given our large dataset, we can revisit this problem and see whether it is now possible to determine which value of β fits the data best.

The gamma comparison model given in (1) is static in that the strengths of the competitors do not change over time. However, in many applications, time invariant strengths are not appropriate. For example, when modelling in sport, it is unreasonable to assume that team (or player) strengths are constant. Similarly when modelling consumer preferences for brands, changing fashions dictate brand strengths and they are unlikely to be constant over time.

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In response, more recent work on modelling paired comparisons has concentrated on extending models to allow for time-varying, or dynamic, paired comparisons. For example, Glickman (1999) presents an approximate Bayesian updating algorithm for estimating dynamic strength parameters of the Bradley–Terry model. The parameters evolve stochastically and he uses the model to construct a list of the best chess players of all time, together with a rankings table of the then current tennis players. Knorr-Held (2000) presents a methodology, based on a model that is more general than the Bradley–Terry model, to estimate stochastically evolving team strengths.

All previous work on dynamic paired-comparisons models employs a stochastic evolution of the strength parameters. There are however, situations when one can justify modelling the evolution of strength parameters deterministically. For individual sports such as tennis, where players compete as individuals, there is clearly a strong systematic component to the evolution of their strength. Typically, as we shall see, a player's strength increases in the early part of their career, reaches a peak and then slowly declines until the player withdraws from competing. We believe it is more appropriate to model this strength so that it varies smoothly over a player's career and hence a deterministic evolution is applicable. Undoubtedly a stochastic element is present, for example when a player is injured. However, injured players normally stop competing until largely recovered. In either case, the contribution of the deterministic component to strength evolution surely outweighs that of the stochastic element.

When modelling in individual sports, we therefore propose using a paired comparisons model that allows for a deterministic evolution of player strengths. In this paper we first present a closed form for Stern's gamma comparison model and then extend it to allow for time-varying strength parameters for each competitor utilising barycentric rational interpolants. We choose to demonstrate our model using a rich and interesting data set: the results of tennis matches from men's professional tennis Grand Slam tournaments since 1968, and look for an answer to the question: who is the greatest player of all-time?

The paper is organised as follows: Section 2 presents a closed form for Stern's gamma comparison model. Section 3 describes our time-varying paired comparisons model with two tennis-motivated extensions to the model given in Section 4. Results from fitting the model to tennis data are given in Section 5, and Section 6 concludes the paper with some closing remarks.

2. A closed form for Stern's gamma model for paired comparisons

Stern (1990) presents a model that generalises the BT and TM paired comparisons models in two aspects: first, it incorporates both the BT and TM models and all models inbetween, and second, it generalises paired comparisons to the case of multiple comparisons, of which the Plackett–Luce model (Plackett, 1975) is a notable member. In this paper we consider the case of paired comparisons only and show that, for this special case of Stern's model, that it is possible to obtain the probability that player i wins as the distribution function of a symmetric beta distribution. We generalise Stern's model slightly first, to allow players i and j to have different shape parameters β_i and β_j. The result we use to derive our closed form parameterisation is that the ratio of two gamma-distributed random variables follows a modified beta distribution (see Johnson, Kotz, & Balakrishnan, 1994).

On standardising the random variables in (1), we have

$$p_{ij} = \int_0^\infty \int_0^{\alpha_i/\alpha_j s} \frac{z^{\beta_i-1}\exp(-z)s^{\beta_j-1}\exp(-s)\,dz\,ds}{\Gamma(\beta_i)\Gamma(\beta_j)}.$$

Now set $z = ts$; the Jacobian of the transformation is s, so

$$p_{ij} = \int_0^\infty \int_0^{\alpha_i/\alpha_j} \frac{t^{\beta_i-1}s^{\beta_i+\beta_j-1}\exp(-(1+t)s)\,dt\,ds}{\Gamma(\beta_i)\Gamma(\beta_j)}.$$

We now change the order of integration, which is allowed, as Fubini's theorem is satisfied. Integrating over s we obtain

$$p_{ij} = \frac{\Gamma(\beta_i+\beta_j)}{\Gamma(\beta_i)\Gamma(\beta_j)} \int_0^{\alpha_i/\alpha_j} \frac{t^{\beta_i-1}\,dt}{(1+t)^{\beta_i+\beta_j}},$$

and changing variable to $y = t/(1+t)$ so that $t = y/(1-y)$ and $dt/dy = (1-y)^{-2}$ we obtain

$$p_{ij} = \frac{\Gamma(\beta_i+\beta_j)}{\Gamma(\beta_i)\Gamma(\beta_j)} \int_0^{\alpha_i/(\alpha_i+\alpha_j)} y^{\beta_i-1}(1-y)^{\beta_j-1}\,dy, \qquad (2)$$

i.e. the probability that player i beats player j is the distribution function of the beta distribution. For Stern's model, $\beta_i = \beta_j \equiv \beta$ so that p_{ij} is the distribution function of the symmetric beta distribution. This form of Stern's model is excellent for computation, as the incomplete beta ratio is a special function that is widely available from software platforms such as fortran, MatLab or R. We believe that analysts fitting paired comparisons models could adopt this generalised model in (2), rather than choosing between the BT or TM model.

We note that it would be possible to fit an individual shape parameter β for each player, so departing from the class of linear models, for which F must be a symmetric distribution, i.e. $F(-x) = 1 - F(x)$. However, for the present we restrict ourselves to a common value of β for all players.

3. A time-varying paired comparisons model

Our approach to allow for time-varying strengths, such that $\alpha_i \to \alpha_i(t)$ in (2), is to estimate strength parameters at each of several nodes for every player and then interpolate between the nodes to obtain values of a player's strength at any point during his career. The choice of the number of nodes is described below. The obvious methodology to adopt for the interpolation would be spline interpolation. However, we find that barycentric rational interpolants (BRI) (Floater & Hormann, 2007; Press, Teukolsky, Vetterling, & Flannery, 2007) provide a more accurate fit in general, and have the added advantage of being simpler.

The time-varying strength is modelled using the barycentric rational interpolant, so that the strength of player i at time t is given by

$$\alpha_i(t) = \frac{\sum_{k=1}^{n_i} w_{ik}\lambda_{ik}/(t-t_{ik})}{\sum_{k=1}^{n_i} w_{ik}/(t-t_{ik})} \qquad (3)$$

where λ_{ik} is the kth fitted strength of player i, i.e. the strength at time t_{ik}. There are n_i such nodes for player i, and we discuss the weights w_{ik} next.

Berrut (1988) gave weights for a barycentric rational interpolant, where a ratio of polynomials is used to interpolate between the points λ_{ik}. Berrut's interpolant was free of poles, a potential problem with ratios of polynomials. This approach was developed further by Floater and Hormann (2007) who derived sets of weights giving accuracies of h^{d+1} where d is the order of the interpolant and h the largest step size. The weights of order zero are $w_k = (-1)^k$, and weights of order one are

$$w_k = (-1)^k\left(\frac{1}{t_k - t_{k-1}} + \frac{1}{t_{k+1} - t_k}\right),$$

with terms with out-of-range values x_0 or x_{n+1} omitted. For equally-spaced values of y_{ik} the weights are again $w_k = (-1)^k$, but now the end nodes have weight halved. This resembles the halving of

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